SYMBOLIC ALGEBRA OR THE ALGEBRA OF ALGEBRAIC NUMBERS: TOGETHER WITH CRITICAL NOTES ON THE METHODS OF REASONING EMPLOYED IN GEOMETRY

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SYMBOLIC ALGEBRA

OR

THE ALGEBRA OF ALGEBRAIC NUMBERS,

Together with Critical Notes on the Methods of Reasoning Employed in Geometry.

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PREFACE.

The object of the following essay is the discussion of negative quantities as found in algebra, or rather the finding a logically developed system, which shall include such quantities as special cases, and thus tend to the great generalization of problems and theorems.

In the older algebras the fundamental theorems were established on the supposition that the various letters, used as symbols, denoted pure *arithmetical* or *absolute* numbers, and the results were *assumed* to be general or true for the so-called negative quantities as well as ordinary numbers. But there are so many objections to a method without a rational basis, even though it gives true results, admitting of a proper interpretation, that of late years attempts have been made to substitute an algebraic series of numbers, involving both plus and minus quantities, for the ordinary series, so that a definite meaning may be given to a minus number, and the ordinary algebraic processes be given a rational basis for all cases. We shall endeavor in what follows to deduce the ordinary laws of addition, subtraction, multiplication and division, for such a series, calling especial attention to difficulties which naturally occur to any one in the development of the common system, and pointing out how these difficulties are overcome by the "symbolical algebra," as it has been termed.

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1. It will conduce to a better appreciation of the subject if we first give very briefly the usual method of deducing the ground rules for arithmetical numbers and point out their limitations.

We shall first then regard the letters used as symbols to denote ordinary numbers.

2. *Addition.*—Let it be required to add $(8a-9b)$ to $(5b-4a)$.

It is tacitly assumed that $(8a-9b)$ is positive and denotes a real number, also that $(5b-4a)$ is positive and denotes an ordinary arithmetical number, whole or fractional. That is, we assume that if the proper numerical values are substituted for a and b, that $8a$ is greater than $9b$, and that $5b$ is numerically greater than $4a$.

Now, if we simply add $5b$ to $(8a-9b)$, we have $(5b+8a-9b)$, but this sum is too great by $4a$, as we had to add $5b$ diminished by $4a$, so that we must subtract $4a$ from this first result to get the correct sum, which is therefore,

$$5b+8a-9b-4a.$$

In order to reduce this expression to its fewest terms, we have to make use of the *law*, that is easily proved for numbers, that the order in which we combine the terms is immaterial. Thus, if from $(5b+8a)$ we subtract $9b$, the result is $(8a-4b)$, since we evidently reach the same value by adding $5b$ to $8a$ and then subtracting $9b$, as in simply subtracting $4b$ from $8a$. From $8a-4b$, we have now to subtract $4a$, giving $4a-4b$ for the correct answer.

In practice we set down the terms and add thus:

$$\begin{array}{r} 8a-9b \\ -4a+5b \\ \hline 4a-4b \end{array}$$

But it must be distinctly understood that $-4a$ by itself means nothing, and that we have only written for convenience like terms under each other, so that $(-4a+5b)$ must be interpreted $(5b-4a)$, which is agreeable to the law mentioned.

From a consideration of such examples we deduce the law in addition: Combine like terms by adding those of like sign and prefixing the common sign, and when of unlike signs take the difference of the sum of the positive and the sum of the negative terms and prefix the sign of the greater.

3. *Subtraction.*—Suppose we have to subtract

$$5b-4a \text{ from } 8a-9b.$$

If we take $5b$ from the minuend, the indicated result is

$$8a-9b-5b;$$

but we have taken away too much by $4a$, for we had only to subtract $5b$ less $4a$, so that we must increase this result by $4a$, giving for the correct answer

$$8a-9b-5b+4a=12a-14b.$$

Combining the terms as mentioned above by adding $8a$ and $4a$ and subtracting

14b, which is the same thing as first subtracting 9b and then 5b. This result is briefly represented thus:

$$\begin{array}{rl} \text{From} & 8a-9b \\ \text{Subtract} & -4a+5b \\ \hline \text{Remainder} = & 12a-14b \end{array}$$

which is thus equivalent to the rule; change the signs of the subtrahend and proceed as in addition.

We again note that the minuend and subtrahend are tacitly assumed to be real positive numbers, whole or fractional, and further that the subtrahend is numerically in value less than the minuend. There is, besides, no sense in subtracting —4a from +8a by itself, for —4a has no existence by itself. Neither can we subtract 5b from —9b since the last term is an absurdity by itself in arithmetical algebra. Neither can any meaning be attached to adding the same terms (—4a to 8a or 5b to —9b), in article 2, though we get so accustomed to using the well-known rules in apparently adding or subtracting such single terms that