ON THE PARTITIONING OF REGULAR NETWORKS

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by

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ABSTRACT

Partitions of regular interconnection networks used for multiprocessing are investigated. Bounds on the number of components as a function of the number of connections between components are given. The relation between the existence of partitions for networks and their ability to support efficiently data motions is examined.

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1. INTRODUCTION

The fast advance in microelectronics has fostered interest in new architectures suited to this technology. In particular, much research has been devoted to the design of systems consisting of large, regular networks of identical computation nodes. The advance in VLSI has also motivated a large number of papers on the theoretical constraints of VLSI technology. In particular, the area required to realize different types of regular communication graphs have been investigated.

The silicon area required to realize a large computational system frequently exceeds the area of one chip. The network has to be packaged into several components. The cost of multichip systems is best reflected by the number of components rather than by the sum of their area. The "component number" complexity of a network is therefore no less important than its area complexity.

The number of components needed to realize a given network is affected by two constraints. The "area" of a component, i.e. number of nodes that can be packed on a component, is restricted by the physical size of the chip. The "perimeter" of a component, i.e. number of lines leaving that component, is restricted by the pin count of the chip. The last constraint becomes increasingly important with the advance in VLSI technology. The pin count of VLSI chips increase more slowly than the gate count since the size of external wires does not scale down at the same rate as the internal feature size. Also, there is an increasing disparity between the speed of internal lines and the speed of external lines. This limits the use of time-multiplexing to overcome the pin count constraint. It is therefore important to study the effect of "perimeter" constraints on the partionability of interconnection graphs.

Such a study is undertaken in this paper. we characterize the "area" to "perimeter" relationship for families of common graphs such as grides, trees, and shuffle-exchange graphs. We proceed next to relate the partitionability of graphs to their ability to support efficiently data motions, and show that networks which are efficient for routing do not partition well.

2. DEFINITIONS

We represent a network by a triple $N = \langle V,P,E \rangle$, where $\langle V \cup P,E \rangle$ is an undirected multigraph (multiple edges between nodes are allowed). V is the set of (internal) <u>nodes</u> of N, E is the set of <u>edges</u> of N, and P is the set of <u>ports</u> of N. Ports will be used to represent external connections to the network. We say that the network is <u>closed</u> if $P = \emptyset$, <u>open</u> otherwise.

Let U be a subset of V. We define the <u>boundary</u> of U, to be the set of edges connecting nodes from U to nodes outside U. The <u>size</u> s(U) of U, is the number of nodes in U, and the <u>perimeter</u> p(U) of U is the number of edges in the boundary of U.

Π is a <u>partition</u> of the network N if it is a partition of the internal nodes of N. A partition of N is an <u>(s,p)-partition</u> if each component has size at most s and perimeter at most p. We can associate with each partition Π of the network N a network $\Pi(N)$ defined as follows: The ports of $\Pi(N)$ are the ports of N; the nodes of $\Pi(N)$ are the components of the partition; and the edges of $\Pi(N)$ are the edges of N connecting nodes in different components. Thus, N admits an (s,p)-partition with k components iff there exist a graph homomorphism $f:N \to N'$ such that

1. f defines a one to one correspondence between the ports of N and the ports of N'.
2. N' has k internal nodes.
3. For each internal node v of N' $|f^{-1}(v)| \leq s$ and degree(v) $\leq p$.

3. MESH-CONNECTED NETWORKS

A d-dimensional mesh-connected network of size $M = m^d$ consists of the nodes $\langle \alpha_1 \ldots \alpha_d \rangle$, $1 \leq \alpha_i \leq m$. The node $\langle \alpha_1 \ldots \alpha_d \rangle$ is connected to the nodes $\langle \alpha_1 \ldots \alpha_j \pm 1 \ldots \alpha_d \rangle$, $1 \leq j \leq d$, where such node exists. In an open mesh-connected network the nodes at the boundary of the mesh (i.e nodes $\langle \alpha_1 \ldots \alpha_d \rangle$ such that $\alpha_i = 1$ or $\alpha_i = m$ for some i) are assumed to be ports. Of particular interest are 2-dimensional grids which have simple planar layouts. With such layout the perimeter of a set of nodes is related to the perimeter of a surface, whereas the number of nodes in the set is related to its area. We can therefore bound the maximal size of a set as a function of its perimeter.

We recall the following facts from planar geometry.

FACT 1. The largest area of a surface circumscribed by a curve of fixed length is achieved by a circle. Thus, the area A of a surface is related to its perimeter P by the inequality $A \leq P^2/4\pi$. The same inequality is still valid when the surface is the union of several connected components.

FACT 2. The largest area A of a surface circumscribed by a straight segment and a curve of length P is achieved by a half-circle, and is $P^2/2\pi$.

FACT 3. The largest area A of a surface circumscribed by two orthogonal segments and a curve of length P is achieved by a quadrant, and is P^2/π.

THEOREM 3.1 Let U be a subset of internal nodes in a 2-dimensional open mesh-connected network. Then $s(U) = O(p(U)^2)$.

PROOF: We represent the network by a square grid, with nodes at unit distances. We associate to each node P in U a unit square centered at P. The union of these squares forms a surface S with area s(U) and perimeter p(U) (Fig. 1). The claim now follows from fact 1.

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CORROLARY 3.2 A minimal (∞,p)-partition of a 2-dimentional open mesh connected network of size M has $\Theta(M/p^2)$ components.

PROOF: The lower bound follows from the previous theorem. An (s,p)-partition of a 2-dimensional mesh-connected network into components of size $\Theta(p^2)$ is simply achieved by by partitioning into subsquares with side p.

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The same argument can be carried through for d-dimensional mesh-connected networks, $d > 2$, by considering regular grids in d-dimensional space. We obtain:

THEOREM 3.3 (i) The maximal size of a component with perimeter p in an open d-dimensional mesh-connected networks is $\Theta(p^{d/(d-1)})$.

(ii) A minimal (∞,p) partition of a d-dimensional open mesh connected network of size M has $\Theta(M/p^{d/(d-1)})$ components.

Note that the constant implicit in the Θ notation depends on d.

The previous asymptotic results are valid for closed mesh-connected networks as well. Let N be a closed 2-dimensional mesh-connected network of size M and let U be a connected subset of nodes of size $s \leq \alpha M$, for some constant $\alpha < 1$, and perimeter p. If each connected component of the complement of U is supported by at most two sides of N then we have (using fact 3) $M - s \leq p^2/\pi$, so that

$$(1-\alpha)M \leq \frac{p^2}{\pi}, \text{ and}$$

$$s \leq \alpha M \leq \frac{\alpha}{\pi(1-\alpha)}p^2.$$

If the boundary of U reaches accross two opposite sides of N then we have $p \geq M^{1/2}$, and $s \leq p^2$. Otherwise U is supported by at most two sides of N, so that

$$s \leq p^2/\pi.$$

These inequalities imply the following theorem.

THEOREM 3.4 (i) Let $\alpha < 1$ be a fixed constant. The maximal size of a component