TABLES OF [SQUARE ROOT OF] 1-R2 AND 1-R2 FOR USE IN PARTIAL CORRELATION AND IN TRIGONOMETRY

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TABLES OF $\sqrt{1-r^2}$ AND $1-r^2$

FOR USE IN

PARTIAL CORRELATION AND IN TRIGONOMETRY

BY

JOHN RICE MINER, Sc. D.

Associate in the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, The Johns Hopkins University

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TABLES OF $\sqrt{1-r^2}$ AND $1-r^2$ FOR USE IN PARTIAL CORRELATION AND IN TRIGONOMETRY[1]

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INTRODUCTION

In the calculation of partial correlation coefficients each coefficient is obtained from those of the next lower order by the formula:

$$r_{12\cdot 34\cdots n}=\frac{r_{12\cdot 34\cdots(n-1)}-r_{1n\cdot 34\cdots(n-1)}\,r_{2n\cdot 34\cdots(n-1)}}{\sqrt{1-r^2_{1n\cdot 34\cdots(n-1)}}\sqrt{1-r^2_{2n\cdot 34\cdots(n-1)}}}. \qquad \text{(i)}$$

A large part of the labor of calculation is thus involved in the determination of the factors in the denominator of the formula, and may be saved by reference to a table of $\sqrt{1-r^2}$ for different values of r. Similarly, a standard deviation of higher order is calculated from the formula:

$$\sigma_{1\cdot 23\cdots n}=\sigma_{1\cdot 23\cdots(n-1)}\sqrt{1-r^2_{1n\cdot 23\cdots(n-1)}} \qquad \text{(ii)}$$

in which again labor may be saved by reference to such a table, which I have calculated at the suggestion of Dr. Raymond Pearl.

Table I gives the values of $\sqrt{1-r^2}$ for $r=.0001$ to $.9999$, for values of the argument proceeding by differences of .0001. A similar table, but on a much less extended scale, has been given by Holbrook Working, in Quarterly Publications of the American Statistical Association, XVII, 767, June, 1921. Since $\sin^2 a+\cos^2 a=1$, the table may also be used to obtain $\cos a$ when $\sin a$ is given and vice versa.

Table II, which gives the values of $1-r^2$ on a more extended scale than Table VIII of Pearson's Tables for Statisticians and Biometricians, may be used in the calculation of the probable error of the correlation coefficient from the formula:

$$PE_r=\frac{.67449}{\sqrt{n}}\,(1-r^2)=\chi_1(1-r^2). \qquad \text{(iii)}$$

[1] Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 55.

In calculating the tables the value of $1-r^2$ was obtained for each value of r and its square root then calculated by dividing $1-r^2$ by an approximate value of the square root and taking the mean of the divisor and quotient. This method of obtaining square roots depends on the fact that $\frac{x^2}{x+\delta}=x-\delta+\frac{\delta^2}{x}-\ldots..$ If, therefore, δ is small as compared with x the quotient will be nearly equal to $x-\delta$ and the mean of divisor and quotient will be a much closer approximation to x than the original $x+\delta$. Where, as in the greater part of this table, a close approximation to the square root can be found by differences from the preceding items, this method is very convenient. All work was done on a calculating machine and each table checked by differences.

In arrangement the tables are in general like logarithmic tables. The column at the left of the page gives the first three figures of r, while the fourth figure is given in the heading. To find the value of $\sqrt{1-r^2}$ for a given value of r, we therefore look in Table I for the first three figures of r (including zeros) in the column at the left of the page headed r. In the same line and in the column headed by the fourth figure of r we find the last figures of $\sqrt{1-r^2}$. For $r=.0000$ to $.7000$ the first three figures of $\sqrt{1-r^2}$ are given as the separated figures in the column headed 0. Should there be no separated figures in column 0 in this line the three standing next above should be taken, except when the first of the figures in the column headed by the fourth figure of r has a bar above it, when we must take the separated figures from the line below in column 0. For $r=.7000$ to $.8500$ the first three figures of $\sqrt{1-r^2}$ are given in column 0 and column 5. If, therefore, the fourth figure of r is between 0 and 4, inclusive, the first three figures of $\sqrt{1-r^2}$ are to be taken from column 0 in the same line, except when there is a bar over the first figure in the column headed by the fourth figure of r, in which case the first three figures of $\sqrt{1-r^2}$ are found in column 5 in the same line. If the fourth figure of r is between 5 and 9, inclusive, the first three figures of $\sqrt{1-r^2}$ are to be taken from column 5 in the same line, except when there is a bar over the first figure in the column headed by the fourth figure of r, in which case the first three figures of $\sqrt{1-r^2}$ are to be taken from column 0 of the line below. For $r=.8500$ to $.9999$ all the figures of $\sqrt{1-r^2}$ are given in each column.

For example, if $r = .0476$ we find the first three figures .047 in the column r on page 7. In the same line and in the column 6 we find the figures 866. The separated figures in the column 0 next above the line are .998, and $\sqrt{1-r^2}$ is therefore .998 866. If $r = .1944$ we find .194 in the column r on page 10. In the same line and in the column 4 we find the figures 922. Since there is a bar over the first figure we take the .980 from the line below in column 0, and $\sqrt{1-r^2}$ is .980 922. If $r = .7432$ we find .743 in the column r on page 21. In the same line in column 2 we find 069 and in column 0 the first three figures .669, so that $\sqrt{1-r^2}$ is .699 069. If $r = .7433$ we find in column 3 of the same line $\bar{9}58$. Since there is a bar over the first figure, we take the .668 from column 5, and $\sqrt{1-r^2}$ is .668 958. If $r = .7757$ we find .775 in column r on page 22. In column 7 of the same line we find 102 and in column 5 the first three figures .631, so that $\sqrt{1-r^2}$ is .631 102. If $r = .7758$ in column 8 of the same line we find $\bar{9}79$. As there is a bar over the first figure we take .630 from column 0 of the line below, and $\sqrt{1-r^2}$ is .630 979. If $r = .8937$ we find .893 in column r on page 25 and in column 7 of the same line find $\sqrt{1-r^2}$ to be .448 665. If r is given to more than four places of decimals the value of $\sqrt{1-r^2}$ may if desired be obtained by the usual methods of interpolation.

Probably the most advantageous form in which to arrange the calculation of partial correlation coefficients is the following:

Coefficient		$\sqrt{1-r^2}$	Product term	Numerator	Denominator	Coefficient	
r_{12}	+.0476	.998 87	+.144 48	−.096 88	.656 30	$r_{12\cdot3}$	−.1476
r_{13}	+.1944	.980 92	+.035 38	+.159 02	.668 31	$r_{13\cdot2}$	+.2379
r_{23}	+.7432	.669 07	+.009 25	+.733 95	.979 81	$r_{23\cdot1}$	+.7491

The derivation of the $\sqrt{1-r^2}$ column from Table I has already been explained. In the product term column +.144 48 is the product of +.1944 and +.7432, while the other items are similarly derived. In the numerator column $-.096\ 88 = (+.0476) - (+.144\ 48)$, while .65630 in the denominator column is the product of .980 92 and .669 07 from the $\sqrt{1-r^2}$ column. In the last column −.1476 is the quotient of −.096 88 divided by .656 30.

The arrangement of Table II is similar to that of Table I, except that for $r = .0000$ to $.5000$ the first three figures of $1 - r^2$ are given in column 0, while for $r = .5000$ to $.9999$ they are given in both column 0 and column 5. To obtain the probable error of r, $1 - r^2$ from Table II is multiplied by χ_1 from Table V of Pearson's Tables for Statisticians and Biometricians. For example, if for a population of 453, $r = -.5627$, we find .562 in column r of Table II on page 41. In column 7 of the same line we find 369. As r is between .5000 and .9999 we look in column 5 for the first three figures .683, so that $1 - r^2$ is .683 369. From Table V we find $\chi_1 = .03169$ for $n = 453$. Therefore, the probable error of $r = .03169 \times .683\ 369 = .0217$.

Table I. Values of $\sqrt{1-r^2}$

r	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
.000	1.000 000	000	000	000	000	000	000	000	000	000
.001	.999 999	999	999	999	999	999	999	999	998	998
.002	998	998	998	997	997	997	997	996	996	996
.003	995	995	995	995	994	994	994	993	993	992
.004	992	992	991	991	990	990	989	989	988	988
.005	987	987	986	986	985	985	984	984	983	983
.006	982	981	981	980	980	979	978	978	977	976
.007	975	975	974	973	973	972	971	970	970	969
.008	968	967	966	966	965	964	963	962	961	960
.009	959	959	958	957	956	955	954	953	952	951
.010	950	949	948	947	946	945	944	943	942	941
.011	939	938	937	936	935	934	933	932	930	929
.012	928	927	926	924	923	922	921	919	918	917
.013	915	914	913	912	910	909	908	906	905	903
.014	902	901	899	898	896	895	893	892	890	889
.015	888	886	884	883	881	880	878	877	875	874
.016	872	870	869	867	866	864	862	861	859	857
.017	855	854	852	850	849	847	845	843	842	840
.018	838	836	834	833	831	829	827	825	823	821
.019	819	818	816	814	812	810	808	806	804	802
.020	800	798	796	794	792	790	788	786	784	782
.021	779	777	775	773	771	769	767	765	762	760
.022	758	756	754	751	749	747	745	742	740	738
.023	735	733	731	729	726	724	721	719	717	714
.024	712	710	707	705	702	700	697	695	692	690
.025	687	685	682	680	677	675	672	670	667	665
.026	662	659	657	654	651	649	646	643	641	638
.027	635	633	630	627	625	622	619	616	614	611
.028	608	606	602	599	597	594	591	588	585	582
.029	579	577	574	571	568	565	562	559	556	553
.030	550	547	544	541	538	535	532	529	526	522
.031	519	516	513	510	507	504	501	497	494	491
.032	488	485	481	478	475	472	468	465	462	459
.033	455	452	449	445	442	439	435	432	429	425
.034	422	418	415	412	408	405	401	398	394	391
.035	387	384	380	377	373	370	366	363	359	355
.036	352	348	345	341	337	334	330	326	323	319
.037	315	312	308	304	300	297	293	289	285	282
.038	278	274	270	266	262	259	255	251	247	243
.039	239	235	231	227	224	220	216	212	208	204
.040	200	196	192	188	184	180	175	171	167	163
.041	159	155	151	147	143	139	134	130	126	122
.042	118	113	109	105	101	096	092	088	084	079
.043	075	071	066	062	058	053	049	045	040	036
.044	032	027	023	018	014	009	005	000	996	991
.045	.998 987	982	978	973	969	964	960	955	951	946
.046	941	937	932	928	923	918	914	909	904	900
.047	895	890	885	881	876	871	866	862	857	852
.048	847	843	838	833	828	823	818	813	809	804
.049	799	794	789	784	779	774	769	764	759	754
.050	749	744	739	734	729	724	719	714	709	704