# A COURSE IN DESCRIPTIVE GEOMETRY AND PHOTOGRAMMETRY FOR THE MATHEMATICAL LABORATORY

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A Course in Descriptive Geometry and Photogrammetry for the Mathematical Laboratory by E. Lindsay Ince

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## E. LINDSAY INCE

# A COURSE IN DESCRIPTIVE GEOMETRY AND PHOTOGRAMMETRY FOR THE MATHEMATICAL LABORATORY



## Edinburgh Mathematical Tracts

No. I

## DESCRIPTIVE GEOMETRY AND PHOTOGRAMMETRY

## A COURSE IN DESCRIPTIVE GEOMETRY AND PHOTOGRAMMETRY

FOR THE

## MATHEMATICAL LABORATORY

BY

E. LINDSAY INCE, M.A., B.Sc.



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## PREFACE

One of the most desirable attainments of the mathematician is the ability to form a mental picture of three dimensional systems with ease and perspicuity, an ability which cannot better be developed than by a course of Descriptive Geometry.

It is a singular and regrettable fact that in most British Universities, Descriptive Geometry has hitherto been omitted from the regular mathematical curriculum and studied only in the technical classes. The tendency of recent years, however, is towards a recognition of its unique value, both from the educational and from the practical point of view.

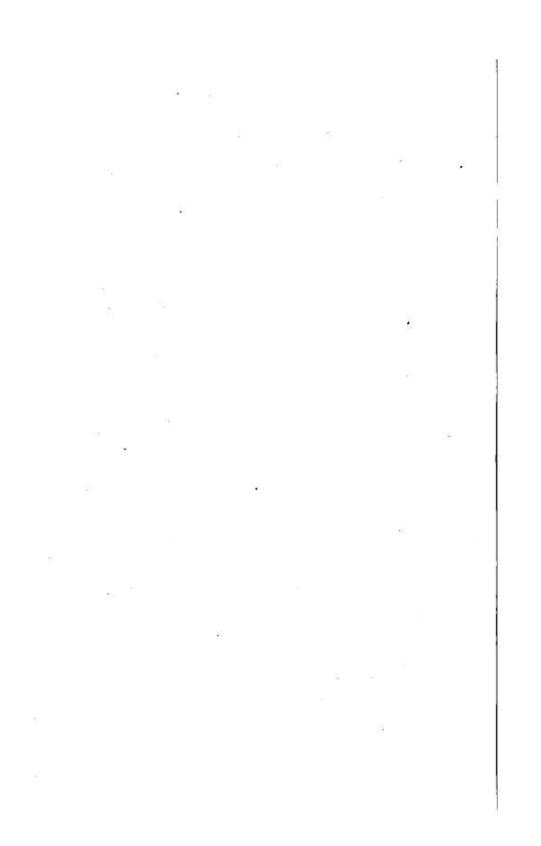
The present tract embodies the course which is given to the non-technical students in the Mathematical Laboratory of the University of Edinburgh. In its compilation I have consulted from time to time the works of Catalan, Antomari, and Gino Loria, and, above all, the Géométrie Descriptive of Monge, which has been a never-failing source of inspiration.

I cannot bring this Preface to a conclusion without expressing my sense of deep obligation to Professor Whittaker for his invaluable suggestions and criticism, both during the compilation of the tract and during its passage through the press.

E. L. I.

THE MATHEMATICAL LABORATORY, University of Edinburgh, 23rd June 1915.

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