

**A TEXT-BOOK ON  
THE METHOD OF  
LEAST SQUARES**

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A text-book on the method of least squares by Mansfield Merriman

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**MANSFIELD MERRIMAN**

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LEAST SQUARES**



A TEXT-BOOK  
ON THE  
METHOD OF LEAST SQUARES.

BY  
MANSFIELD MERRIMAN,  
MEMBER OF AMERICAN MATHEMATICAL SOCIETY.

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## PREFACE TO THE FIRST EDITION.

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The "Elements of the Method of Least Squares," published in 1877, was written with two objects in view: first, to present the fundamental principles and processes of the subject in so plain a manner, and to illustrate their application by such simple and practical examples, as to render it accessible to civil engineers who have not had the benefit of extended mathematical training; and, secondly, to give an elementary exposition of the theory which would be adapted to the needs of a large and constantly increasing class of students.

In preparing the following pages the author has likewise kept these two objects continually in mind. While the former work has been used as a basis, the alterations and additions have been so numerous and radical as to render this a new and distinct book rather than a second edition. The arrangement of the theoretical and practical parts is entirely different. In Chapters I to IV is presented the mathematical development of the principles, methods, and formulas; while in Chapters V to IX the application of these

to the different classes of observations is made, and illustrated by numerous practical examples. For the use of both students and engineers, it is believed that this plan will prove more advantageous than the one previously followed. Hagen's deduction of the law of probability of error is given, as well as that of Gauss. More attention is paid to the laws of the propagation of error, the solution of normal equations, and the deduction of empirical formulas. Many new illustrative examples of the adjustment and comparison of observations have been selected from actual practice, and are discussed in detail. At the end of each chapter are given a few problems or queries; and in the Appendix are eight tables for abridging computations.

MANSFIELD MERRIMAN.

LEHIGH UNIVERSITY, SOUTH BETHLEHEM, PA.

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#### NOTE TO THE EIGHTH EDITION.

The seventh edition was the result of a thorough revision and was enlarged by the addition of new matter on the solution of normal equations, on the uncertainty of the probable error, and on the median. In this edition all known errors have been corrected and an alphabetical index has been added.

M. M.

32 WEST FORTIETH STREET, NEW YORK.



# CONTENTS.

## CHAPTER I.

### *INTRODUCTION.*

CLASSIFICATION OF OBSERVATIONS . . . . .	2
ERRORS OF OBSERVATIONS . . . . .	3
PRINCIPLES OF PROBABILITY . . . . .	6
PROBLEMS . . . . .	12

## CHAPTER II.

### *LAW OF PROBABILITY OF ERROR.*

AXIOMS DERIVED FROM EXPERIENCE . . . . .	13
THE PROBABILITY CURVE . . . . .	15
FIRST DEDUCTION OF THE LAW OF ERROR . . . . .	17
SECOND DEDUCTION OF THE LAW OF ERROR . . . . .	22
DISCUSSION OF THE PROBABILITY CURVE . . . . .	25
THE PROBABILITY INTEGRAL . . . . .	27
COMPARISON OF THEORY AND EXPERIENCE . . . . .	31
REMARKS ON THE FUNDAMENTAL FORMULAS . . . . .	33
PROBLEMS AND QUERIES . . . . .	35

## CHAPTER III.

### *THE ADJUSTMENT OF OBSERVATIONS.*

WEIGHTS OF OBSERVATIONS . . . . .	36
THE PRINCIPLE OF LEAST SQUARES . . . . .	38
DIRECT OBSERVATIONS ON A SINGLE QUANTITY . . . . .	41
INDEPENDENT OBSERVATIONS OF EQUAL WEIGHT . . . . .	43
INDEPENDENT OBSERVATIONS OF UNEQUAL WEIGHT . . . . .	51
SOLUTION OF NORMAL EQUATIONS . . . . .	56
CONDITIONED OBSERVATIONS . . . . .	57
PROBLEMS . . . . .	65

## CHAPTER IV.

*THE PRECISION OF OBSERVATIONS.*

THE PROBABLE ERROR . . . . .	66
PROBABLE ERROR OF THE ARITHMETICAL MEAN . . . . .	70
PROBABLE ERROR OF THE GENERAL MEAN . . . . .	72
LAWS OF PROPAGATION OF ERROR . . . . .	75
PROBABLE ERRORS FOR INDEPENDENT OBSERVATIONS . . . . .	79
PROBABLE ERRORS FOR CONDITIONED OBSERVATIONS . . . . .	86
PROBLEMS . . . . .	87

## CHAPTER V.

*DIRECT OBSERVATIONS ON A SINGLE QUANTITY.*

OBSERVATIONS OF EQUAL WEIGHT . . . . .	88
SHORTER FORMULAS FOR PROBABLE ERROR . . . . .	92
OBSERVATIONS OF UNEQUAL WEIGHT . . . . .	95
PROBLEMS . . . . .	99

## CHAPTER VI.

*FUNCTIONS OF OBSERVED QUANTITIES.*

LINEAR MEASUREMENTS . . . . .	101
ANGLE MEASUREMENTS . . . . .	104
PRECISION OF AREAS . . . . .	106
REMARKS AND PROBLEMS . . . . .	107

## CHAPTER VII.

*INDEPENDENT OBSERVATIONS ON SEVERAL QUANTITIES.*

METHOD OF PROCEDURE . . . . .	109
DISCUSSION OF LEVEL LINES . . . . .	110
ANGLES AT A STATION . . . . .	117
EMPIRICAL CONSTANTS . . . . .	124
EMPIRICAL FORMULAS . . . . .	130
PROBLEMS . . . . .	139

## CHAPTER VIII.

*CONDITIONED OBSERVATIONS.*

THE TWO METHODS OF PROCEDURE . . . . .	141
ANGLES OF A TRIANGLE . . . . .	142
ANGLES AT A STATION . . . . .	143
ANGLES OF A QUADRILATERAL . . . . .	147
SIMPLE TRIANGULATION . . . . .	152
LEVELLING . . . . .	154
PROBLEMS . . . . .	160

## CHAPTER IX.

*THE DISCUSSION OF OBSERVATIONS.*

PROBABILITY OF ERRORS . . . . .	162
THE REJECTION OF DOUBTFUL OBSERVATIONS . . . . .	166
CONSTANT ERRORS . . . . .	169
SOCIAL STATISTICS . . . . .	172
PROBLEMS . . . . .	174

## CHAPTER X.

*SOLUTION OF NORMAL EQUATIONS.*

THREE NORMAL EQUATIONS . . . . .	175
FORMATION OF NORMAL EQUATIONS . . . . .	177
GAUSS'S METHOD OF SOLUTION . . . . .	181
WEIGHTED OBSERVATIONS . . . . .	187
LOGARITHMIC COMPUTATIONS . . . . .	190
PROBABLE ERRORS OF ADJUSTED VALUES . . . . .	195
PROBLEMS . . . . .	198

## CHAPTER XI.

*APPENDIX AND TABLES.*

OBSERVATIONS INVOLVING NON-LINEAR EQUATIONS . . . . .	200
MEAN AND PROBABLE ERROR . . . . .	204
UNCERTAINTY OF THE PROBABLE ERROR . . . . .	206
THE MEDIAN . . . . .	208