A DRILL-BOOK IN ALGEBRA

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GEORGE WILLIAM JONES

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BY

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OF

CORNELL UNIVERSITY.

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PREFACE.

So far as concerns mathematical studies there are always two classes of pupils: those that will use mathematics later in life as one of the tools of their trade—the engineers, architects, accountants, teachers, scientists; those that will not so use it.

To both of these classes the careful study of the elements of geometry and algebra is important; to the first class as laying a sure foundation for work in the higher mathematics and for its professional applications; to the others as giving the power and the habit of exact statement and rigorous proof. In this later day we pride ourselves on our laboratories, in which the pupil comes face to face with the facts and forces of nature—every mathematical recitation-room under an able teacher is a laboratory in logic, and for sound logic there is always an unlimited demand.

This book is for use by the more advanced classes in the high schools and academies and the lower classes in the colleges; and its primary object is to teach young men and women to think. From the beginning the philosophy of the subject is made prominent; and in writing it the author set himself the double task of writing a book whose definitions should be precise and whose proofs should be rigorous, and of writing one so simple that any diligent pupil could read it easily.

But he has not confined himself to definitions and proofs: a large collection of questions and exercises has been added; and for a good understanding of the fundamental principles, and readiness in their use, quite as much reliance is placed on

the questions as on the text. It is hoped that by his effort to answer these questions the pupil will be early taught to think earnestly, to think independently, and to think hard.

Believing that the elements of plane geometry are at least as simple as those of algebra, the author has assumed some knowledge of that subject in pupils using this book, and he has not hesitated to use geometric illustrations where their greater concreteness seemed to give greater clearness.

This book was undertaken as an abridgment of Oliver, Wait, and Jones' Treatise on Algebra; and at first it was hoped that cutting out the more abstruse portions of the text and the harder examples would fit the larger work for the use of the preparatory schools; but after that excision many alterations were found necessary, and in the end the order of topics was changed, new lines of proof were adopted, new questions and examples were prepared, and the whole book was rewritten.

The author is indebted both to the writers on algebra, from whose works he has drawn freely, and to his associates at Cornell University, who have been unsparing in their kindly assistance. In particular he returns thanks to Professor Hathaway, who outlined the discussion of the combinatory properties of the simple arithmetic operations, that of measures and multiples, and that of incommensurable numbers; to Mr. John H. Tanner, who made the selections from the Treatise and prepared the first draft of the copy; to Miss Ida M. Metcalf, who spent half a year in giving form to the text and preparing the questions and exercises; and to Professors Oliver and McMahon, who have read the greater part of the book either in manuscript or in proof.

But with all the care he could exercise, he is conscious that many errors have crept in, and that there are many defects that only use in the class-room can bring to light. He will esteem it a great favor, therefore, if his fellow-teachers will tell him freely what they find wrong either in method or matter, in general plan or detail.

SUGGESTIONS TO TEACHERS.

THE author's aim has been to discuss subjects in the order of their logical dependence, so as to construct a continuous and irrefutable line of argument throughout; but it must be remembered that the logical order is not always the easiest or the most natural, and that not only are the claims of mathematical science to be satisfied, but the wants of the individual immature pupil are to be met. It may often happen, therefore, that deviation from the order of the book will be of advantage; and a subsequent review in the order laid down will then show the drift of the thought and the links in the chain of reasoning.

This point should be early and often impressed upon the pupil: that many simple theorems are stated and proved, not that he may be convinced of their truth, for that conviction can be reached by repeated experiment, but that a firm foundation may be laid for a logical structure of ever-increasing height and complexity. That is, they are not ends in themselves, but only useful tools for future work.

If parts of the text seem too abstruse for his pupils, or the questions too hard, the wise teacher will reserve such parts for a later reading, and he will choose for himself the order in which the topics shall be presented. For example, he may find it well to take up parts of the second chapter before finishing the first, or to set parallel lessons from the two chapters. Theory and practice may thus go hand in hand.

In this book general principles are stated formally, as in text-books on geometry, and illustrations and applications follow; but the living teacher may well reverse this order, and before setting a new topic in the book he may draw out the whole matter from the pupils' own mind, by careful questioning after the Socratic method, first in simple illustrations and then in general principles. Afterwards the pupil may read and explain the text, and answer the questions set down for exercises. The author recognizes the distinction between the office of the text-book and that of the teacher, and he

places the man above the book; but he has been taught by his own experience that a book is very useful. And what should the book contain? A treatise on any subject contains the whole body of knowledge on that subject, well digested and well arranged and indexed, so that the reader may find all that he seeks within its pages; but it need contain no exercises for pupils and no questions. A drill-book is more modest in its aims: it leaves out all that is not necessary to the main purpose; it presents the great principles in due order and in simple language, so that the pupil may read them again and again, and it sets him, under his teacher's guidance, to find out their applications; it suggests to him the best methods of work; it offers him lists of questions on which he may task himself, and grow strong in the exercise; it helps the teacher to cross-examine him; it serves as a standard to both teacher and pupil; and it saves them endless labor in the giving and taking of notes and in the preparation and copying of exercises.

It has been the author's good fortune to have a few bright young people come to him every summer for a more complete preparation in elementary mathematics, and he has thus kept fresh in mind the wants of beginners. He has found these pupils needing a regular and persistent drill both in the statement and proof of the fundamental principles and in their application; and he has written this book to meet their wants. He submits it respectfully to the judgment of his fellow-teachers, only asking that they neither adopt it nor reject it without a very careful examination; for while in most things it follows well-worn lines, in others it makes radical departures from the common usage.

An answer-book (not a key) is in preparation for the convenience of teachers; and the whole list of questions has been printed on cards for use in the class-room.

GEORGE W. JONES.

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CONTENTS.

I. THE PRIMARY OPERATIONS OF ARITHMETIC

II. THE PRIMARY OPERATIONS OF ALGEBRA.

III. SIMPLE EQUATIONS.

IV. MEASURES AND MULTIPLES,