

**ELEMENTS OF
PLANE
TRIGONOMETRY**

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Elements of plane trigonometry by Daniel A. Murray

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DANIEL A. MURRAY

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BY
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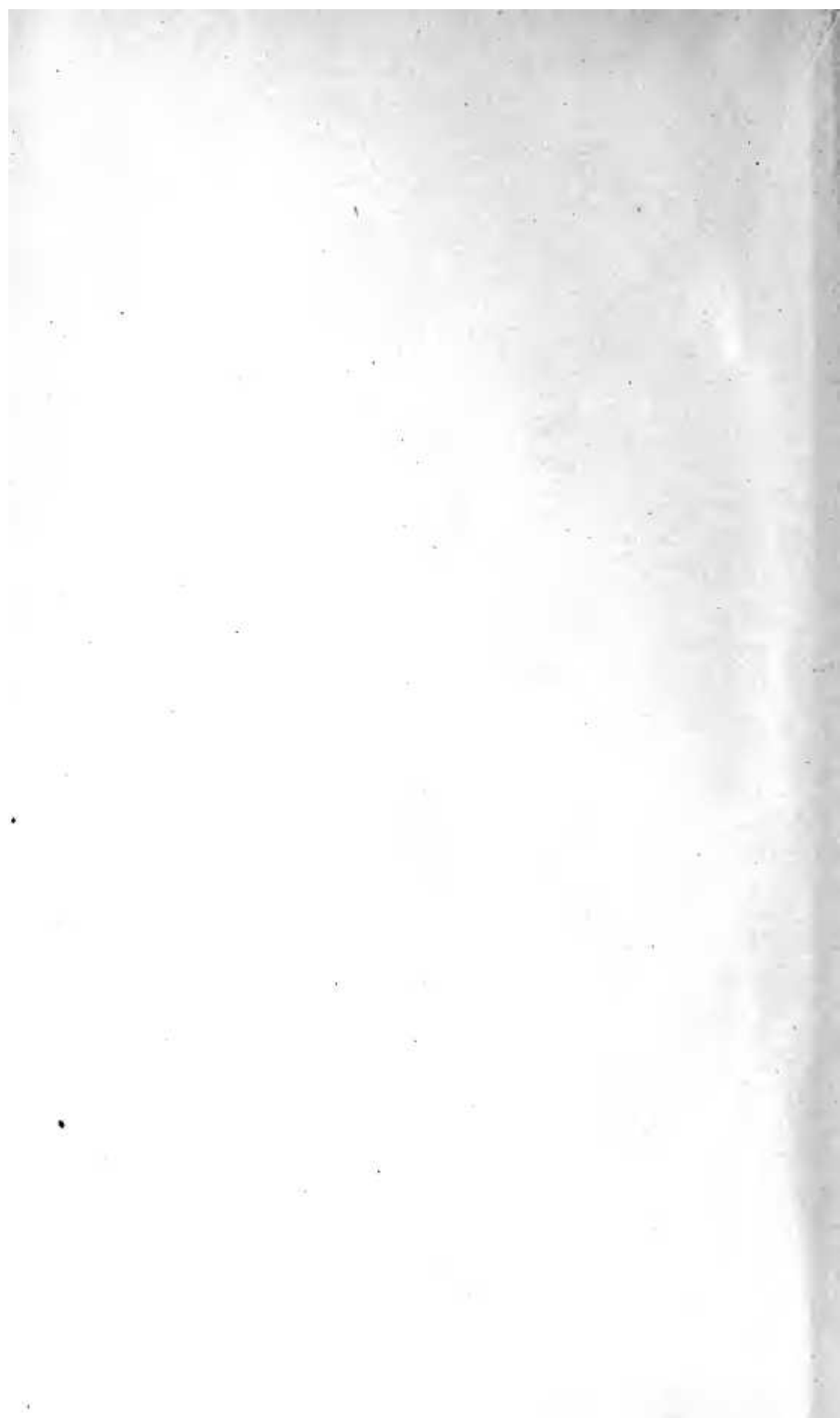
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PREFACE

THIS text-book is shorter than my former text-book, entitled "Plane Trigonometry," by the omission of many of the notes and several of the topics in that book and by the more condensed treatment of other topics. There is also a marked difference in arrangement. Thus, radian measure, the periodicity of the trigonometric functions, their general values, and their graphs, and the inverse trigonometric functions, which are discussed in the later chapters of the "Plane Trigonometry," are treated in the earlier chapters of the "Elements of Plane Trigonometry." The line definitions of the functions are explained more fully, and the unit circle is used to a greater extent, in this book than in the former one.

D. A. MURRAY.

May 1, 1911.



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