

**ON A NEW METHOD OF OBTAINING
THE DIFFERENTIALS OF FUNCTIONS
WITH ESPECIAL REFERENCE TO THE
NEWTONIAN CONCEPTION OF
RATES OR VELOCITIES**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649168767

On a new method of obtaining the differentials of functions with especial reference to the Newtonian conception of rates or velocities by J. Minot Rice & W. Woolsey Johnson

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OR VELOCITIES

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REVISED EDITION

NEW YORK
D. VAN NOSTRAND PUBLISHER
23 MURRAY STREET AND 27 WARREN STREET

1875

PREFACE.

THIS pamphlet is a revised edition of a paper which was read before the American Academy of Arts and Sciences, January 14, 1873, and subsequently published in its Proceedings.

It is now reproduced for the purpose—first, of presenting a new method of deriving the Differentials of Functions by means of their Algebraic characteristics with the aid of a few elementary properties easily established; and, secondly, of showing that the method of rates or fluxions may be advantageously used for the purposes of instruction, and the use of infinitesimals, limits and series entirely avoided until the student is well grounded in the elements of the Calculus; thus securing the advantages afforded by the real and precise definitions of this method, instead of sacrificing them by employing the difficult and readily misconceived notions of limits or infinitesimals in deducing the formulas for differentiation.

In the original paper it was shown that the new method of deducing the differentials was applicable to all the functions of a single variable; some of these applications are now omitted to be replaced by other methods which we consider preferable for the purposes of elementary instruction.

It is our intention to publish a text-book prepared in accordance with this plan, the first part of which has already been printed for the use of the cadets at the U. S. Naval Academy.

J. M. R.

W. W. J.

ANNAPOLIS, MD., *July*, 1875.



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THE NEWTONIAN METHOD OF FLUXIONS.

1. It is a well-known fact that writers on the Differential Calculus deduce the same elementary theorems from fundamental conceptions which have little or no apparent resemblance, and frequently employ methods which present to the conscientious student difficulties of a character too formidable to be ignored.

Notwithstanding the unusual attention which many of the ablest mathematical writers since the time of Newton and Leibnitz have bestowed upon this subject it is undoubtedly true that many instructors still tacitly permit their students to follow the trite precept of D'Alembert,—"*Allez en avant, et la foi vous viendra.*" Few habits are more pernicious to the student of mathematics than that of following rules founded upon principles which he does not thoroughly comprehend; yet it is precisely this habit that D'Alembert's precept tends to confirm. Faith comes—only too soon.

In the words of Condillac—"Ce n'est point par la routine qu'on s'instruit, c'est par sa propre réflexion; et il est essentiel de contracter l'habitude de se rendre raison de ce qu'on fait; cette habitude s'acquiert plus facilement qu'on ne pense; et une fois acquise, elle ne se perd plus."

2. Most of the French writers on the Calculus have adopted a method of treating the subject which may be characterized

as a combination of the method of limits with that of infinitesimals. One of the best examples is the work of J. A. Serret, Paris, 1868. This method, although excellent in extensive treatises like those of Serret and Bertrand, seems to us far too difficult for a beginner not possessed of unusual mathematical ability, especially as it involves several fundamental propositions very hard to comprehend. In nearly all cases, it will be found best for the student not to attempt these works until he has prepared himself by studying one of a less formidable character.

3. A distinguished English writer of mathematical textbooks (Mr. I. Todhunter), who has himself adopted the method of limits, remarks that—"A difficulty of a more serious kind which is connected with the notion of a limit, appears to embarrass many students of this subject, namely, a suspicion that the methods employed are only approximative, and therefore a doubt as to whether the results are absolutely true. This objection is certainly very natural, but at the same time by no means easy to meet, on account of the inability of the reader to point out any definite place at which his uncertainty commences." * This remark seems to us not only to go to the root of the difficulty, but also to suggest an excellent mode of testing the student's comprehension of the subject when taught by the ordinary methods.

* *Todhunter's Differential Calculus*, Macmillan and Co. London, 1860, page 12.