

AN INTRODUCTION TO MATHEMATICAL PHYSICS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649055722

An Introduction to Mathematical Physics by R. A. Houstoun

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

R. A. HOUSTOUN

**AN INTRODUCTION
TO MATHEMATICAL
PHYSICS**

AN INTRODUCTION TO
MATHEMATICAL PHYSICS

AN •INTRODUCTION
TO
MATHEMATICAL PHYSICS •

BY

R. A. HOUSTOUN, M.A., PH.D., D.Sc.

LECTURER ON PHYSICAL OPTICS AND ASSISTANT TO THE PROFESSOR OF
NATURAL PHILOSOPHY IN THE UNIVERSITY OF GLASGOW

UNIVERSITY OF
CALIFORNIA

LONGMANS, GREEN, AND CO.

39 PATERNOSTER ROW, LONDON

NEW YORK, BOMBAY, AND CALCUTTA

1912

All rights reserved

PREFACE

THIS book is the substance of lectures I have given during the past six years to the Natural Philosophy Class A in the University of Glasgow.

It is intended primarily as a class-book for mathematical students and as an introduction to the advanced treatises dealing with the subjects of the different chapters, but since the analysis is kept as simple as possible, I hope it may be useful for chemists and others who wish to learn the principles of these subjects. It is complementary to the text books in dynamics commonly used by junior honours classes.

A knowledge of the calculus and a good knowledge of elementary dynamics and physics is presupposed on the part of the student.

A large proportion of the examples has been taken from examination papers set at Glasgow by Prof. A. Gray, LL.D., F.R.S., to whom I must also express my indebtedness for many valuable suggestions. The proofs have been read with great care and thoroughness by Dr. John McWhan of the Mathematical Department.

R. A. HOUSTOUN.

10

11

12

13

14

15

16

17

18

19

20

21

22

23

24

25

26

CONTENTS

CHAPTER I.

ATTRACTION.

- § 2. Uniform rod at an external point. § 3. Uniform circular disc at a point on its axis. § 5. Homogeneous spherical shell at an external point. § 6. Homogeneous spherical shell at an internal point. § 8. Elliptic homocoid. Internal point. § 9. Attraction at its pole of a homogeneous solid oblate spheroid of small eccentricity. [Examples.] § 10. Theorem of surface integral of normal force. § 11. Solid sphere, the density of which is a function of the distance from the centre. § 12. Infinitely long right circular cylinder, the density being a function of the distance from the axis. § 13. Uniform lamina bounded by two parallel planes and extending to an infinite distance in all directions. § 14. Potential. § 15. Lines of force and equipotential surfaces. § 17. Tubes of force. § 18. Potential due to a homogeneous sphere. § 19. Potential due to infinitely long cylinder. [Examples.] § 20. Gauss's theorem of average potential over a spherical surface. § 21. Gauss's theorem. § 22. Divergence of a vector. § 23. Laplace's and Poisson's equations. § 24. Change of coordinates. § 25. Poisson's equation in polar and cylindrical coordinates. § 26. Example on Poisson's equation. § 27. Electrical images. Point and plane. § 28. Point and sphere. [Examples.]

Pp. 1-28

CHAPTER II.

HYDRODYNAMICS.

- § 30. Acceleration at a point. § 31. Angular velocity at a point. § 32. Curl of a vector. Potential vectors. Stream lines. § 33. Equation of continuity. § 34. Equation of continuity in polar and cylindrical coordinates. § 36. Equations of motion. § 37. Case of impulsive pressure. § 38. Boundary condition. § 39. Green's theorem. § 40. Energy equation. § 41. Integration of the equations of motion. § 42. Bernoulli's theorem. § 43. Applications of Bernoulli's theorem. [Examples.] § 44. Two-dimensional motion. The stream function. § 45. Expression for

the kinetic energy. § 46. Conjugate functions. § 47. Solution of problems in two-dimensional steady irrotational motion. § 48. Application of the method of images. [Examples.] § 49. Motion of a sphere in an infinite liquid. No forces. § 50. Motion of a sphere in an infinite liquid. Gravity acting. [Examples.] § 52. Stokes' theorem. § 54. Kelvin's circulation theorem. § 55. Vortex tubes. § 56. Laws of vortex motion. § 57. The rectilinear vortex. § 60. Kelvin's minimum energy theorem. [Examples.] - - - - - Pp. 29-68

CHAPTER III.

FOURIER SERIES AND CONDUCTION OF HEAT.

§§ 61-65. Fourier series. [Examples.] § 67. Equation for the conduction of heat. § 69. Equation for the conduction of heat in poles and cylindricals. § 70. Boundary conditions. § 71. Uniqueness of solution of problem. § 72. Steady flow in one direction. § 73. Steady flow. Symmetry about a point. § 74. Two dimensions. Steady flow. [Examples.] § 75. Variable linear flow. No radiation. § 76. Equation for variable linear flow with radiation. § 77. Ingenhousz's experiment. § 78. Despretz' formula. § 80. Fourier's ring. § 81. Linear flow in semi-infinite solid. Temperature on face given as harmonic function of the time. § 82. Ångström's method of determining the conductivity of bars. [Examples.] § 83. Flow of heat in a sphere. Surface at zero temperature. § 84. Linear flow in doubly-infinite solid. Fourier's integral. § 85. Other forms of Fourier's integral. § 86. Linear flow in semi-infinite solid. § 87. The age of the earth. § 88. Point source of heat. § 89. Plane source of heat. § 90. Doublets. § 91. Two and three-dimensional Fourier series and integrals. [Examples.] Pp. 69-108

CHAPTER IV.

WAVE MOTION.

§ 92. Transverse vibrations of a stretched string. § 94. Harmonic waves. § 95. String of length l . § 96. Damping. § 97. Energy of a vibrating string. § 98. Longitudinal vibrations in a rod. § 99. Torsional vibrations in a right circular cylinder. [Examples.] § 100. Tidal waves. § 101. Condition for long waves. § 102. Stationary waves in a rectangular trough. § 103. Effect of an arbitrary initial disturbance. § 105. Energy of a harmonic long wave. § 106. Forced waves in a canal. § 107. Gravity waves. General case. § 108. Two horizontal dimensions. Stationary waves in a rectangular vessel. [Examples.] § 109. Sound waves in a gas. § 110. Transverse waves. [Examples.] Pp. 109-135

CHAPTER V.

ELECTROMAGNETIC THEORY.

- § 112. Magnetic potential due to a small magnet. § 113. Magnetic shell. Magnetic potential due to a uniform shell. § 114. Ampère's theorem. § 115. Work done in carrying unit pole round closed path in field of current. § 116. Case of a right circular cylindrical conductor. § 117. First circuital theorem. More general form. § 118. The displacement current. [Examples.] § 119. Current induction. § 120. Currents induced in a mass of metal. § 122. Electromagnetic waves. § 123. Hertz's experiments. § 124. Hertz's theory of the electric doublet. § 125. Poynting's theorem. § 126. Application of Poynting's theorem. § 127. Propagation of a plane wave. § 128. Energy of a plane wave. § 129. Boundary conditions. § 130. Reflection and refraction. § 131. Perpendicular incidence. § 132. Total reflection. § 133. Absorbing media. [Examples.] - - - - - Pp. 136-167

CHAPTER VI.

THERMODYNAMICS.

- § 136. Watt's indicator diagram. § 139. Carnot's cycle. § 140. Application of the second principle of thermodynamics. § 141. Carnot's function. § 142. Kelvin's scale of absolute temperature. § 143. Entropy. § 144. Transformation of thermal coefficients. § 145. Carnot's function. Otherwise. § 148. The perfect gas. § 147. Clapeyron's formula. § 150. Further properties of a perfect gas. § 151. Work done by a perfect gas in a Carnot cycle. § 152. Entropy of a perfect gas. [Examples.] § 153. The porous plug experiment. § 154. Van der Waal's equation. § 155. Effect of pressure on the freezing point. § 156. The specific heat of saturated vapour. § 157. Change of temperature produced in a wire by stretching it. § 158. Effect of temperature on the *e.m.f.* of a reversible cell. § 159. Second definition of entropy. § 160. The second principle of thermodynamics. [Examples.] - - - - - Pp. 168-197

INDEX - - - - - Pp. 198, 199