A COMPANION TO ANY ELEMENTARY WORK ON PLANE TRIGONOMETRY

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A COMPANION TO ANY ELEMENTARY WORK ON PLANE TRIGONOMETRY

Trieste

A COMPANION

TO ANY ELEMENTARY WORK ON

PLANE TRIGONOMETRY:

BUT MORE ESPECIALLY TO THAT OF

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(A.)

$$\text{Sin. A . Cosec. A} = 1 \quad \ldots\ldots \quad 1$$

$$\text{Cos. A . Sec. A} = 1 \quad \ldots\ldots \quad 2$$

$$\text{Tan. A . Cot. A} = 1 \quad \ldots\ldots \quad 3$$

$$\text{Sin.}^2 \text{A} + \text{Cos.}^2 \text{A} = 1 \quad \ldots\ldots \quad 4$$

$$\text{Sec.}^2 \text{A} - \text{Tan.}^2 \text{A} = 1 \quad \ldots\ldots \quad 5$$

$$\text{Cosec.}^2 \text{A} - \text{Cot.}^2 \text{A} = 1 \quad \ldots\ldots \quad 6$$

$$\text{Tan. A} = \frac{\text{sin. A}}{\text{cos. A}} \quad \ldots\ldots \quad 7$$

$$1 - \text{Cos. A} = \text{Vers. A} \quad \ldots \quad 8$$

(1). The following relations are obvious from the above formulæ, simply by dividing—

$$\text{Sin. A} = \frac{1}{\text{cosec. A}}; \ \text{cos. A} = \frac{1}{\text{sec. A}};$$

$$\text{tan. A} = \frac{1}{\text{cot. A}}; \ \text{cot. A} = \frac{\text{cos. A}}{\text{sin. A}}.$$

$$\text{Sin. A} = \sqrt{1 - \cos.^2 \text{A}}\text{ , and cos. A} = \sqrt{1 - \sin.^2 \text{A}}\text{ .}$$

$$\text{Sec. A} = \sqrt{1 + \tan.^2 \text{A}}\text{ , and tan. A} = \sqrt{\sec.^2 \text{A} - 1}\text{ .}$$

$$\text{Cosec. A} = \sqrt{1 + \cot.^2 \text{A}}\text{ , and cot. A} = \sqrt{\text{cosec.}^2 \text{A} - 1}\text{ .}$$

(2). Trigonometrical formulæ can be frequently simplified by reducing them from fractional to integral forms; for this purpose, the formulæ in Art. (1) are of great use.

Reduce $\frac{1}{\sin. \text{A} \cdot \cos. \text{A} (1 - \text{vers. A})}$ to an integral form.

Since $\text{cosec. A} = \frac{1}{\sin. \text{A}}$, and $\sec. \text{A} = \frac{1}{\cos. \text{A}}$,

and $1 - \text{vers. A} = 1 - 1 + \cos. \text{A} = \cos. \text{A}$, we have

$$\frac{1}{\sin. \text{A} \cos. \text{A} (1 - \text{vers. A})} = \frac{1}{\sin. \text{A}} \frac{1}{\cos.^2 \text{A}} = \text{cosec. A} \sec.^2 \text{A}.$$

the integral form required. (See Jeane's Trig., p. 6.)

Transform the following fractions to integral forms :—

1. $\frac{1}{\sin.^2 \text{A} \cos.^2 \text{A}}$. 2. $\frac{1}{\sin. \text{A} (1 - \text{vers. A})}$.

3. $\frac{1}{\cos.^2 \text{A} \sqrt{1 - \cos.^2 \text{A}}}$. 4. $\frac{1}{\sin. \text{A} \sqrt{1 - \sin.^2 \text{A}}}$.

5. $\dfrac{\sin. A}{(1 - \text{vers. } A)\ \sqrt{1 - \cos.^2 A}}$.

6. $\dfrac{\cos. A}{\sin.^2 A \sqrt{1 - \cos.^2 A}\ \sqrt{1 - \sin.^2 A}}$.

(3). Reduce $\dfrac{1}{\tan. A\ .\ \cot.^2 A \sqrt{\text{cosec.}^2 A - 1}}$ to an integral form.

Since $\cot. A = \dfrac{1}{\tan. A}$, and $\tan. A = \dfrac{1}{\cot. A}$, and

$\sqrt{\text{cosec.}^2 A - 1} = \cot. A$;

$\therefore \dfrac{1}{\tan. A\ .\ \cot.^2 A\ \sqrt{\text{cosec.}^2 A - 1}} = \cot. A \tan^2 A \tan. A = \tan.^2 A.$

Transform the following fractions into integral forms :—

1. $\dfrac{\tan. A}{\cot.^3 A, \sec. B \sqrt{1 + \cot.^2 B}}$.

2. $\dfrac{\sec. A}{\sec.^2 A, \text{cosec.}^2 B \sqrt{1 + \tan.^2 A}}$.

3. $\dfrac{\text{cosec. } A}{(1 - \text{vers. } A)^2 \sqrt{1 + \cot.^2 B}}$.

4. $\dfrac{\sin. A, \cos. A}{\sec.^2 A, \text{cosec.}^2 A \sqrt{1 - \sin.^2 A}}$.

5. $\dfrac{\sin.^2 A\,(1 - \text{vers.}\,B)}{(1 - \text{vers.}\,B)^2\,\sqrt{1 + \cot.^2 B}}$.

6. $\dfrac{\cos.^2 A\,\sqrt{1 - \sin.^2 A}}{\sec.\,A\,\tan.\,A\,\cot.^2 A\,\sqrt{\sec.^2 A - 1}}$.

ANSWERS TO ART. (2).

1. Cosec.^{2} A . sec.^{2} A.
2. Cosec. A . sec. A.
3. Sec.^{2} A . cosec. A.
4. Cosec. A . sec. A.
5. Sec. A.
6. Cosec.^{3} A.

ANSWERS TO ART. (3).

1. Tan.^{4} A, cos. B, sin. B.
2. Cos.^{2} A, sin.^{2} B.
3. Cosec. A, sec.^{2} A, sin. B.
4. Sin.^{2} A, cos.^{2} A.
5. Sin.^{2} A, sec. B, sin. B.
6. Cos.^{4} A.

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(4). If sin. A $= \frac{1}{3}$, find the cos. A, tan. A, sec. A.

$$\text{Cos. } A = \sqrt{1 - \sin.^2 A} = \sqrt{1 - \frac{1}{9}} = \frac{2}{3}\sqrt{2}.$$

$$\text{Tan. } A = \frac{\sin.\,A}{\cos.\,A.} = \frac{1}{3} \times \frac{3}{2\sqrt{2}} = \frac{\sqrt{2}}{4}.$$

$$\text{Sec. } A = \sqrt{1 + \tan.^2 A} = \sqrt{1 + \frac{1}{8}} = \frac{3}{2\sqrt{2}}.$$

Given, $\frac{\text{sin. A}}{\text{sin. } x} = \text{cos. A}$, to find cosec. x, sin. x, tan x. (Jeane's Trig., p. 8. Q. 22.)

From the given eq. $\frac{\text{sin. A}}{\text{cos. A}} = \text{sin. } x$;

$$\therefore \text{sin. } x = \text{tan. A}.$$

But, $\text{cosec. } x = \frac{1}{\text{sin. } x} = \frac{1}{\text{tan. A}} = \text{cot. A}$;

$$\therefore \text{tan. } x = \frac{\text{sin. } x}{\text{cos. } x} = \frac{\text{tan. A}}{\sqrt{1 - \text{sin.}^2 x}} = \frac{\text{tan. A}}{\sqrt{1 - \text{tan.}^2 \text{A}}}.$$

Given, $\frac{\text{sin. A cos. B}}{\text{cosec. } x} = \frac{\text{cos. C sin. B}}{\text{tan. D}}$. (Jeane's Trig. p. 9. Q. 25.) Find sin. x.

$$\text{Since sin. } x = \frac{1}{\text{cosec. } x};$$

$$\therefore \text{sin. } x = \frac{\text{cos. C sin. B}}{\text{tan. D sin. A cos. B}} = \text{cosec. A tan. B cos. C. cot. D}.$$

(5). Given, $\frac{\text{sin. A}}{\text{cos. } x} = \frac{\text{cos. A}}{\text{sin. } x}$, find tan. x, sin. x, cos. x.

Since, $\frac{\text{sin. A}}{\text{cos. } x} = \frac{\text{cos. A}}{\text{sin. } x}$; $\therefore \frac{\text{sin. } x}{\text{cos. } x} = \frac{\text{cos. A}}{\text{sin. A}}$;

$$\therefore \text{tan. } x = \text{cot. A}.$$

But $\cos. x = \frac{1}{\sec. x} = \frac{1}{\sqrt{1 + \tan.^2 x}} = \frac{1}{\sqrt{1 + \cot.^2 A}}$,

and $\sin. x = \frac{1}{\operatorname{cosec.} x} = \frac{1}{\sqrt{1 + \cot.^2 x}} = \frac{1}{\sqrt{1 + \tan.^2 A}}$.

The sec. x, and cosec. x, and cot. x, may always be readily obtained from the Eqs., Art. (1).

Solve the following:

1. Given, $\frac{\sec. A}{\operatorname{cosec.} x} = \frac{\operatorname{cosec.} A}{\sec. x}$. Find tan. x, sin. x, and cos. x.

2. Given, $\frac{\tan. A}{\cot. x} = \frac{\cot. A}{\tan. x}$. Find tan. x, sin. x, and cos. x.

3. Given, $\frac{\operatorname{cosec.} A}{\sec. x} = \frac{\sec. A}{\operatorname{cosec.} x}$. Find tan. x, sin. x, and cos. x.

4. Given, $\frac{1}{\sin. x} = 4$. Find cos. x, sin. x, tan. x.

ANSWERS.

1. Cot. A, $\frac{1}{\sqrt{1 + \tan.^2 A}}$, $\frac{1}{\sqrt{1 + \cot.^2 A}}$.

2. $\pm$ cot. A, $\frac{1}{\sqrt{1 + \tan.^2 A}}$, $\frac{1}{\sqrt{1 + \cot.^2 A}}$.

3. The same as in (1).

4. $\frac{\sqrt{15}}{4}$, $\frac{1}{4}$, $\frac{1}{\sqrt{15}}$.