ELEMENTS OF GEOMETRY

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OF

PLANE GEOMETRY:

BY

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PREFACE.

Most persons do not possess, and do not easily acquire, the power of abstraction requisite for apprehending the Geometrical conceptions, and for keeping in mind the successive steps of a continuous argument. Hence, with a very large proportion of beginners in Geometry, it depends mainly upon the *form* in which the subject is presented whether they pursue the study with indifference, not to say aversion, or with increasing interest and pleasure.

In compiling the present treatise, this fact has been kept constantly in view. All unnecessary discussions and scholia have been avoided; and such methods have been adopted as experience and attentive observation, combined with repeated trials, have shown to be most readily comprehended. No attempt has been made to render more intelligible the simple notions of position, magnitude, and direction, which every child derives from observation; but it is believed that these notions have been limited and defined with mathematical precision.

A few symbols, which stand for words and not for operations, have been used, but these are of so great utility in giving *style* and *perspicuity* to the demonstrations that no apology seems necessary for their introduction.

Great pains have been taken to make the page attractive. The figures are large and distinct, and are placed in the middle of the page, so that they fall directly under the eye in immediate connection with the corresponding text. The *given* lines

of the figures are full lines, the lines employed as *aids* in the demonstrations are short-dotted, and the *resulting* lines are long-dotted.

In each proposition a concise statement of what is given is printed in one kind of type, of what is required in another, and the demonstration in still another. The reason for each step is indicated in small type between that step and the one following, thus preventing the necessity of interrupting the process of the argument by referring to a previous section. The number of the section, however, on which the reason depends is placed at the side of the page. The constituent parts of the propositions are carefully marked. *Moreover, each distinct assertion in the demonstrations, and each particular direction in the constructions of the figures, begins a new line; and in no case is it necessary to turn the page in reading a demonstration.*

This arrangement presents obvious advantages. The pupil perceives at once what is given and what is required, readily refers to the figure at every step, becomes perfectly familiar with the language of Geometry, acquires facility in simple and accurate expression, rapidly *learns to reason*, and lays a foundation for the complete establishing of the science.

A few propositions have been given that might properly be considered as corollaries. The reason for this is the great difficulty of convincing the average student that any importance should be attached to a corollary. Original exercises, however, have been given, not too numerous or too difficult to discourage the beginner, but well adapted to afford an effectual test of the degree in which he is *mastering* the subjects of his reading. Some of these exercises have been placed in the early part of the work in order that the student may discover, at the outset, that to commit to memory a number of theorems and to reproduce them in an examination is a useless and pernicious labor; but to learn their uses and applications, and to acquire a readiness in exemplifying their utility, is to derive the full benefit of that mathematical training which looks not so much to the

attainment of information as to the *discipline of the mental faculties.*

It only remains to express my sense of obligation to Dr. D. F. Wells for valuable assistance, and to the University Press for the elegance with which the book has been printed; and also to give assurance that any suggestions relating to the work will be thankfully received.

G. A. WENTWORTH.

Phillips Exeter Academy,
January, 1878.

NOTE TO THIRD EDITION.

In this edition I have endeavored to present a more rigorous, but not less simple, treatment of Parallels, Ratio, and Limits. The changes are not sufficient to prevent the simultaneous use of the old and new editions in the class; still they are very important, and have been made after the most careful and prolonged consideration.

I have to express my thanks for valuable suggestions received from many correspondents; and a special acknowledgment is due from me to Professor C. H. Judson, of Furman University, Greenville, South Carolina, to whom I am indebted for assistance in effecting many improvements in this edition.

TO THE TEACHER.

When the pupil is reading each Book for the first time, it will be well to let him write his proofs on the blackboard in his own language; care being taken that his language be the simplest possible, that the arrangement of work be vertical (without side work), and that the figures be accurately constructed.

This method will furnish a valuable exercise as a language lesson, will cultivate the habit of neat and orderly arrangement of work, and will allow a brief interval for deliberating on each step.

After a Book has been read in this way the pupil should review the Book, and should be required to draw the figures free-hand. He

should state and prove the propositions orally, using a pointer to indicate on the figure every line and angle named. He should be encouraged, in reviewing each Book, to do the original exercises; to state the converse of propositions; to determine from the statement, if possible, whether the converse be true or false, and if the converse be true to demonstrate it; and also to give well-considered answers to questions which may be asked him on many propositions.

The Teacher is strongly advised to illustrate, geometrically and arithmetically, the principles of limits. Thus a rectangle with a constant base b, and a variable altitude x, will afford an obvious illustration of the axiomatic truth contained in [4], page 88. If x increase and approach the altitude a as a limit, the area of the rectangle increases and approaches the area of the rectangle $a\,b$ as a limit; if, however, x decrease and approach zero as a limit, the area of the rectangle decreases and approaches zero for a limit. An arithmetical illustration of this truth would be given by multiplying a constant into the approximate values of any repetend. If, for example, we take the constant 60 and the repetend .3333, etc., the approximate values of the repetend will be $\frac{3}{10}$, $\frac{33}{100}$, $\frac{333}{1000}$, $\frac{3333}{10000}$, etc., and these values multiplied by 60 give the series 18, 19.8, 19.98, 19.998, etc., which evidently approach 20 as a limit; but the product of 60 into $\frac{1}{3}$ (the limit of the repetend .333, etc.) is also 20.

Again, if we multiply 60 into the different values of the decreasing series, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, etc., which approaches zero as a limit, we shall get the decreasing series, 2, $\frac{1}{5}$, $\frac{1}{50}$, $\frac{1}{500}$, etc.; and this series evidently approaches zero as a limit.

In this way the pupil may easily be led to a complete comprehension of the whole subject of limits.

The Teacher is likewise advised to give frequent written examinations. These should not be too difficult, and sufficient time should be allowed for accurately constructing the figures, for choosing the best language, and for determining the best arrangement.

The time necessary for the reading of examination-books will be diminished by more than one-half, if the use of the symbols employed in this book be permitted.

G. A. W.

PHILLIPS EXETER ACADEMY,
January, 1879.

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