A COLLECTION OF EXAMPLES ON HEAT AND ELECTRICITY

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A collection of examples on heat and electricity by H. H. Turner

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BY

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PREFACE.

THESE Examples are collected from the Examination Papers set in the various College Examinations and the Mathematical Tripos. Having had some trouble in obtaining a sufficient number of problems while working for Section D of Part III. of the Mathematical Tripos, I thought others might find it convenient to have them collected into this form.

TRINITY COLLEGE,
June, 1884.

EXAMPLES ON HEAT AND ELECTRICITY.

HEAT.

Fourier's Theorem.

1. Shew that the equation

$$y=\frac{a}{2}+x-\frac{4a}{\pi^2}\left\{\cos\frac{\pi}{a}(x+y)+\frac{1}{3^2}\cos\frac{3\pi}{a}(x+y)+\frac{1}{5^2}\cos\frac{5\pi}{a}(x+y)+\&c.\right\}$$

represents a staircase of straight lines of length a, starting from the origin and parallel alternately to axes of y and x.

S. John's College, 1881.

2. Shew that $\frac{2}{\pi}\int_0^\infty \sin qx\left\{\frac{h}{q}+\tan\alpha\frac{\sin qb-\sin qa}{q^2}\right\}dq$ is the ordinate of a broken line running parallel to the axis of x from $x=0$ to $x=a$ and from $x=b$ to $x=\infty$ and inclined to the axis of x at an angle α between $x=a$ and $x=b$.

Tripos, 1883.

3. Shew that for all values of x between $-b$ and b

$$F(x)-F(-x)=\frac{2}{\pi}\int_0^\infty \sin xu\,du\int_{-b}^{+b}F(y)\sin uy\,dy.$$

S. John's College, 1881.

4. Assuming the truth of Fourier's series prove the formula

$$f(x)=\frac{2}{\pi}\int_0^\infty\int_0^\infty f(v)\cos ux\cos uv\,du\,dv,$$

where x is positive, and state any other necessary conditions. If $f(x)$ is an even function, prove

$$\left(\cosh\frac{d}{dx}\right)^n f(x)=\frac{2}{\pi}\int_0^\infty\int_0^\infty f(v)\cos ux\cos^n u\cos uv\,du\,dv.$$

Tripos, 1881.

5. Assuming Fourier's Integral

$$f(x, y) = \frac{1}{4\pi^2} \iiiint f(\xi, \eta) \cos \{\alpha (\xi - x) + \beta (\eta - y)\} \, d\xi \, d\eta \, d\alpha \, d\beta,$$

shew that in polar co-ordinates

$$f(r, \theta) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\infty} \int_0^{\infty} f(\rho, \phi) F(\gamma, \rho') \, d\phi \, \rho \, d\rho \, \gamma \, d\gamma,$$

where ρ' is the distance from (ρ, ϕ) to (r, θ) and

$$F(\omega) = \frac{1}{2\pi} \int_0^{2\pi} \cos (\omega \cos \lambda) \, d\lambda$$

(i.e. a Bessel's Function).

Tripos, 1880.

6. Let $\phi_1(x)$ be the integral of $\phi(x)$ taken from $x = 0$ to $x = x$ and let it be such that $\phi_1(x)$ never exceeds a certain maximum value: and let $\int_{-\infty}^{\infty} \frac{\phi_1(x)}{x} dx$ have a value A, which is neither zero nor infinite. Prove the following generalization of Fourier's Theorem

$$f(x) = \frac{1}{2A} \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} \phi(zy - zx) f(y) \, dy,$$

and deduce Fourier's result.

(Liouville.)

Conduction of Heat.

One Dimensional Problems.

7. A square prismatic bar has its ends maintained at constant temperatures, one T^0 above and the other t^0 below that of the surrounding air. If a section which divides the bar into segments whose lengths are as 2 : 1 remains at the temperature of the air, compare T and t.

Trinity Hall, 1878.

8. A uniform bar (length $2l$) of small section with an adiathermanous jacket is heated so that the three successive portions of length $\frac{2l}{3}$ are respectively at the temperatures v_1, v_2, v_3; shew that after a time