

**A COLLECTION OF  
EXAMPLES ON HEAT  
AND ELECTRICITY**

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A collection of examples on heat and electricity by H. H. Turner

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## PREFACE.

THESE Examples are collected from the Examination Papers set in the various College Examinations and the Mathematical Tripos. Having had some trouble in obtaining a sufficient number of problems while working for Section D of Part III. of the Mathematical Tripos, I thought others might find it convenient to have them collected into this form.

TRINITY COLLEGE,

*June, 1884.*





## EXAMPLES ON HEAT AND ELECTRICITY.

### HEAT.

#### *Fourier's Theorem.*

1. Shew that the equation

$$y = \frac{\alpha}{2} + x - \frac{4\alpha}{\pi^2} \left\{ \cos \frac{\pi}{\alpha} (x+y) + \frac{1}{3^2} \cos \frac{3\pi}{\alpha} (x+y) + \frac{1}{5^2} \cos \frac{5\pi}{\alpha} (x+y) + \&c. \right\}$$

represents a staircase of straight lines of length  $\alpha$ , starting from the origin and parallel alternately to axes of  $y$  and  $x$ .

S. John's College, 1881.

2. Shew that  $\frac{2}{\pi} \int_0^{\infty} \sin qx \left\{ \frac{x}{q} + \tan \alpha \frac{\sin qb - \sin qa}{q^2} \right\} dq$  is the ordinate of a broken line running parallel to the axis of  $x$  from  $x=0$  to  $x=a$  and from  $x=b$  to  $x=\infty$  and inclined to the axis of  $x$  at an angle  $\alpha$  between  $x=a$  and  $x=b$ .

Tripos, 1883.

3. Shew that for all values of  $x$  between  $-b$  and  $b$

$$F(x) - F(-x) = \frac{2}{\pi} \int_0^{\infty} \sin xu \, du \int_{-b}^{+b} F(y) \sin uy \, dy.$$

S. John's College, 1881.

4. Assuming the truth of Fourier's series prove the formula

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(v) \cos ux \cos uv \, du \, dv,$$

where  $x$  is positive, and state any other necessary conditions. If  $f(x)$  is an even function, prove

$$\left( \cosh \frac{\alpha}{2x} \right)^n f(x) = \frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(v) \cos ux \cosh^n u \cos uv \, du \, dv.$$

Tripos, 1881.

## 5. Assuming Fourier's Integral

$$f(x, y) = \frac{1}{4\pi^2} \iiint f(\xi, \eta) \cos \{a(\xi - x) + \beta(\eta - y)\} d\xi d\eta da d\beta,$$

shew that in polar co-ordinates

$$f(r, \theta) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \int_0^\infty f(\rho, \phi) F(\gamma, \rho') d\phi \rho d\rho \gamma d\gamma,$$

where  $\rho'$  is the distance from  $(\rho, \phi)$  to  $(r, \theta)$  and

$$F(\omega) = \frac{1}{2\pi} \int_0^{2\pi} \cos(\omega \cos \lambda) d\lambda$$

(i.e. a Bessel's Function).

Tripos, 1880.

6. Let  $\phi_1(x)$  be the integral of  $\phi(x)$  taken from  $x=0$  to  $x=x$  and let it be such that  $\phi_1(x)$  never exceeds a certain maximum value : and let  $\int_{-\infty}^{\infty} \frac{\phi_1(x)}{x} dx$  have a value  $A$ , which is neither zero nor infinite. Prove the following generalization of Fourier's Theorem

$$f(x) = \frac{1}{2A} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} \phi(xy - zx) f(y) dy,$$

and deduce Fourier's result.

(Liouville.)

## CONDUCTION OF HEAT.

*One Dimensional Problems.*

7. A square prismatic bar has its ends maintained at constant temperatures, one  $T^0$  above and the other  $T^0$  below that of the surrounding air. If a section which divides the bar into segments whose lengths are as 2 : 1 remains at the temperature of the air, compare  $T$  and  $t$ .

Trinity Hall, 1878.

8. A uniform bar (length  $2l$ ) of small section with an adathermal jacket is heated so that the three successive portions of length  $\frac{2l}{3}$  are respectively at the temperatures  $v_1, v_2, v_3$ ; shew that after a time