A NEW PROBLEM IN HYDRODYNAMICS WITH EXTRANEOUS FORCES ACTING

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WITH

EXTRANEOUS FORCES ACTING

by

EDWARD LEE HANCOCK

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The solution of most problems in Hydrodynamics depends upon the proper combination of the equations of motion of the fluid interior of a given closed surface with the differential equation of the surface, or with the equations expressing the boundary conditions.

Lord Kelvin has shown that the differential equation of the surface for both compressible and incompressible fluids has the following form:

$$u.F'(x) + v.F'(y) + w.F'(z) + F'(t) = 0$$

where (t) is a variable parameter of the equation

$$F(x, y, z, t) = 0$$

In the treatment of problems of the motion of incompressible fluids in three dimensions, where the surface under discussion is spherical or nearly so, the usual particular solutions of Laplace's equation ($\nabla^2\phi = 0$), such as, Zonal, Tesseral and Spherical Harmonics, are adequate, since in these cases the velocity-potential satisfies Laplace's equation. The solution used in any particular case depends upon the symmetry of the boundary conditions. Where the surface differs much from the spherical form as in ellipsoids,

Ellipsoidal Harmonics are used. Problems of this kind have been extensively investigated.

In discussing the anchor ring Mr. W. M. Hicks[(1)] has derived modified forms of the Zonal, Tesseral and Spherical Harmonics by means of which the potential both outside and inside the ring may be completely investigated. The same problem has been solved by Mr. F. W. Dyson[(2)] by using elliptic integrals.

The problem is much simplified when the motion takes place in a single plane in which case, if the boundary consists of a straight line, two parallel straight lines, or is rectangular, the velocity-potential may be expressed as a Fourier's Series or a Fourier's Integral.

In other cases there is no direct method of procedure. The inverse process of finding what boundary conditions will give known solutions of Laplace's Equation is used, with the hope of finding the desired solution. The method of

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(1) Phil. Trans. 1893.

(2) Phil. Trans. 1881, Part III.

images is also applicable to some cases, more especially perhaps in the case of rotational motion.

For the irrotational motion of a perfect liquid there always exists a velocity-potential which satisfies the equation

$$\nabla^2\phi = 0$$

The potential ϕ and the rectangular velocities u, v and w may be found from the given conditions, for all points of the interior. The potential being always least at the boundary the lines of flow and equi-potential lines begin and end there. This is true whether the motion is "steady" or not and true therefore when the extraneous force is gravity.

Much work has been done on the motion of many of the regular solids immersed in a liquid, when acted upon by a system of impulsive forces and also by constant forces. The motions of the liquid in the neighborhood of such solids has also been discussed. Both tidal waves and waves due to local causes have been investigated and their properties discussed to some extent. The related problem of the effect of high land masses upon neighboring bodies of water has been