

**ELEMENTS OF THE
DIFFERENTIAL CALCULUS,
WITH EXAMPLES AND
APPLICATIONS**

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Elements of the differential calculus, with examples and applications by W. E. Byerly

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W. E. BYERLY

**ELEMENTS OF THE
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ELEMENTS
OF THE
DIFFERENTIAL CALCULUS,
WITH
EXAMPLES AND APPLICATIONS.

A TEXT BOOK

BY

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PREFACE.

THE following book, which embodies the results of my own experience in teaching the Calculus at Cornell and Harvard Universities, is intended for a text-book, and not for an exhaustive treatise.

Its peculiarities are the rigorous use of the Doctrine of Limits as a foundation of the subject, and as preliminary to the adoption of the more direct and practically convenient infinitesimal notation and nomenclature; the early introduction of a few simple formulas and methods for integrating; a rather elaborate treatment of the use of infinitesimals in pure geometry; and the attempt to excite and keep up the interest of the student by bringing in throughout the whole book, and not merely at the end, numerous applications to practical problems in geometry and mechanics.

I am greatly indebted to Prof. J. M. Peirce, from whose lectures I have derived many suggestions, and to the works of Benjamin Peirce, Todhunter, Duhamel, and Bertrand, upon which I have drawn freely.

W. E. BYERLY.

CAMBRIDGE, *October, 1879.*

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