

**CAMBRIDGE TRACTS IN MATHEMATICS AND
MATHEMATICAL PHYSICS. NOS.: 2. THE
INTEGRATION OF FUNCTIONS OF A SINGLE
VARIABLE; 12. ORDERS OF INFINITY. THE
'INFINITÄRCALCÜL' OF
PAUL DU BOIS-REYMOND; 18. THE GENERAL
THEORY OF DIRICHLET'S SERIES**

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G. H. HARDY & MARCEL RIESZ

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THE
INTEGRATION OF FUNCTIONS
OF A SINGLE VARIABLE

by

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PREFACE

THIS tract has been long out of print, and there is still some demand for it. I did not publish a second edition before, because I intended to incorporate its contents in a larger treatise on the subject which I had arranged to write in collaboration with Dr Bromwich. Four or five years have passed, and it seems very doubtful whether either of us will ever find the time to carry out our intention. I have therefore decided to republish the tract.

The new edition differs from the first in one important point only. In the first edition I reproduced a proof of Abel's which Mr J. E. Littlewood afterwards discovered to be invalid. The correction of this error has led me to rewrite a few sections (pp. 36-41 of the present edition) completely. The proof which I give now is due to Mr H. T. J. Norton. I am also indebted to Mr Norton, and to Mr S. Pollard, for many other criticisms of a less important character.

G. H. H.

January 1916.

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