

A SIMPLE LAMBDA- CALCULUS MODEL OF PROGRAMMING LANGUAGES

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A simple lambda-calculus model of programming languages by S. Kamal Abdali

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S. KAMAL ABDALI

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PROGRAMMING
LANGUAGES**

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Courant Institute of
Mathematical Sciences

AEC Computing and Applied Mathematics Center

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of Programming Languages

S. Kamal Abdali

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ABSTRACT

We present a simple correspondence between a large subset of ALGOL 60 language and lambda-calculus. With the aid of this correspondence, a program can be translated into a single lambda-expression. In general, the representation of a program is specified by means of a system of simultaneous conversion relations among the representations of the program constituents. High-level programming language features are treated directly, not in terms of the representations of machine-level operations. The model includes input-output in such a way that when the representation of a (convergent) program is applied to the input item representations, the resulting combination reduces to a tuple of the representations of the output items. This model does not introduce any imperative notions into the calculus; the descriptive programming constructs, such as expressions, and the imperative ones, such as assignments and jumps, are translated alike into pure lambda-expressions. The applicability of the model to the problems of proving program equivalence and correctness is illustrated by means of simple examples.

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1. INTRODUCTION

If one wishes to study properties of programs, then one should either develop rules of deduction and inference that apply directly to programming language constructs (e.g., Hoare [1]), or represent programs by the objects of some mathematical system [e.g., 2,3,7-10] and work with these representations. As a step in the second direction, this paper describes a way of representing programs by lambda-expressions [4-6].

Since a number of lambda-calculus (or, related, combinatory logic) models of programming languages have already appeared in the literature [among them, 7-10], the proposal of yet another such model may require justification. So we will first indicate some distinguishing features of our model vis-à-vis others.

1. Our model does not introduce any imperative or otherwise foreign notions to lambda-calculus. This is in contrast to Landin [7], in which imperative features of programming languages are accounted for by ad hoc extensions of the calculus. We find that the calculus, in its purity, suffices as a model of programming languages. By not making any additions to the lambda-calculus, we have the guarantee that all its properties, in particular, the consistency and the Church-Rosser property [6], are valid in our model. For example, even when a program requires a fixed order of execution, the lambda-expression obtained by evaluating the program representation in any order, whatsoever, represents the program result correctly.

2. Programs are translated into lambda-expressions, not interpreted by a lambda-calculus interpreter (Reynolds [10]). Thus, programming semantics is completely reduced to lambda-calculus semantics, but without commitment to any particular view of the latter. Also, all lambda-expression transformations are applicable to programs.

3. Assignments are modelled by the substitution operation of lambda-calculus. Consequently, the notions of memory, addresses, and fetch and store operations do not enter our model in an explicit manner (Stratchey [8], Reynolds [10]).