WENTWORTH & HILL'S. EXAMINATION MANUALS. NO. II. ALGEBRA

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G. A. WENTWORTH & G. A. HILL

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No. II.

ALGEBRA.

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PREFACE.

THIS Manual consists of two parts: The first part contains one hundred and fifty examination papers, the questions for which have been selected mainly from the best English, French, and German collections of problems. These papers may be divided into three groups. The first fifty papers embrace the subjects of Elementary Algebra as far as Quadratic Equations; the next fifty papers also include Quadratic Equations and Radical Expressions; the last fifty papers extend over still more ground, including several topics usually regarded as belonging to Higher Algebra.

In each of these groups the earlier papers will be found somewhat easier than the later ones. The papers are intended to be *hour* papers, but if any of them are thought to be too long for one hour, the time may be increased or the length of the paper reduced by omitting one or more of the questions.

The second part of the Manual is a collection of recent papers actually set in various American and English institutions of learning.

There are two ways in which the Manual may be used.

First: To *test* the learner's knowledge in the usual way by means of an examination. For this purpose the class will come to the recitation-room provided with the Manual and blank books, and the teacher will simply designate by number the paper to be worked.

Secondly: To *exercise* the learner from day to day in the various rules and processes, to detect his weak points, and ascer-

tain where he most needs assistance. This may be done by assigning exercises to be worked in the class-room, or by assigning to each member of the class a paper with directions to hand in the solutions, neatly worked out, at a subsequent recitation.

The Manual will be found especially useful in reviewing the subject of Algebra, and in preparing for examinations.

Answers to the problems in the first one hundred and fifty papers, bound separately in paper covers, can be had by teachers *only*, on application to the publishers.

G. A. WENTWORTH.
G. A. HILL

SPECIMEN PAPER WORKED OUT.

1. Simplify

$$a-[2b+\{3c-3a-(a+b)\}+\{2a-(b+c)\}]$$

$$=a-[2b+\{3c-3a-a-b\}+\{2a-b-c\}]$$
$$=a-[2b+3c-3a-a-b+2a-b-c]$$
$$=a-2b-3c+3a+a+b-2a+b+c$$
$$=3a-2c.$$

Ans. $3a-2c$.

2. Resolve into factors

$$x^2-y^2+z^2-a^2-2xz+2ay$$

$$=(x^2-2xz+z^2)-(a^2-2ay+y^2)$$
$$=(x-z)^2-(a-y)^2$$
$$=\{(x-z)-(a-y)\}\{(x-z)+(a-y)\}$$
$$=(x-z-a+y)(x-z+a-y).$$

3. Find the H.C.F. of

$$5x^2(12x^3+4x^2+17x-3)$$
and $$10x(24x^3-52x^2+14x-1).$$

$5x^2(12x^3+4x^2+17x-3)$	$10x(24x^3-52x^2+14x-1)$	Reserve $5x$
$12x^3+4x^2+17x-3$	$24x^3-52x^2+14x-1$	2
$12x^3+4x^2-x$	$24x^3+8x^2+34x-6$	
$18x-3$	$-60x^2-20x+5$	
$6x-1$	$12x^2+4x-1$	x
	$12x^2-2x$	$2x+1$
	$6x-1$	
	$6x-1$	

Ans. $5x(6x-1)$.

4. Simplify $\frac{3}{1-2x} - \frac{7}{1+2x} - \frac{4-20x}{4x^2-1}$

$= -\frac{3}{1-2x} - \frac{7}{1+2x} - \frac{4-20x}{4x^2-1}$

$= -\frac{3}{1-2x} - \frac{7}{1+2x} + \frac{4-20x}{1-4x^2}$.

L.C.D. $= 1-4x^2$.

$3+6x$ = first numerator,

$-7+14x$ = second numerator,

$4-20x$ = third numerator.

0 = sum of numerators. *Ans.* 0.

5. Solve

$10x+15y-24z = 41$ (1)

$15x-12y+16z = 10$ (2)

$18x-14y-7z = -13$. (3)

Multiply (1) by 2, $20x+30y-48z = 82$

Multiply (2) by 3, $45x-36y+48z = 30$

Add, $65x-6y = 112$ (4)

Multiply (2) by 7, $105x-84y+112z = 70$

Multiply (3) by 16, $288x-224y-112z = -208$

Add, $393x-308y = -138$ (5)

Multiply (4) by 154, $10010x-924y = 17248$

Multiply (5) by 3, $1179x-924y = -414$

Subtract, $8831x = 17662$

$\therefore x = 2$.

Substitute value of x in (4), $130-6y = 112$.

$\therefore y = 3$.

Substitute values of x and y in (1), $20+45-24z = 41$.

$\therefore z = 1$.

Ans. $x = 2$, $y = 3$, $z = 1$.

6. A passenger train, after travelling an hour, is detained 15 minutes; after which it proceeds at three-fourths of its former rate, and arrives 24 minutes late. If the detention had taken place 5 miles farther on, the train would have been only 21 minutes late. Determine the usual rate of the train.

Let x = usual rate of train per hour,

and y = number of miles train has to run.

Then $y - x$ = number of miles train has to run after detention,

$\frac{y-x}{x}$ = number of hours usually required to run $y-x$ miles.

and $\frac{y-x}{\frac{4}{5}x}$ = number of hours actually required to run $y-x$ miles,

Since the detention was 15 minutes, and the train was 24 minutes late, the loss in running-time is 9 minutes, or $\frac{3}{20}$ of an hour.

$$\therefore \frac{y-x}{\frac{4}{5}x} - \frac{y-x}{x} = \frac{3}{20} \qquad (1)$$

If the detention had occurred 5 miles farther on, the loss in running-time would have been 6 minutes, or $\frac{1}{10}$ of an hour.

$$\therefore \frac{y-x-5}{\frac{4}{5}x} - \frac{y-x-5}{x} = \frac{1}{10} \qquad (2)$$

Simplify (1), $20y - 29x = 0$

Simplify (2), $20y - 26x = 100$

Subtract (1) from (2), $3x = 100$

$\therefore x = 33\frac{1}{3}$.

Ans. $33\frac{1}{3}$ miles.

7. Solve

$$\frac{1}{2x^2+x+1} + \frac{1}{2x^2-3x+1} = \frac{a}{2bx-b} - \frac{2bx+b}{ax^2-a},$$

$$\frac{1}{(2x-1)(x+1)} + \frac{1}{(2x-1)(x-1)} = \frac{a}{b(2x-1)} - \frac{2bx+b}{a(x-1)(x+1)}.$$

L.C.D. $= ab(x-1)(x+1)(2x-1)$.

Simplify, $abx - ab + abx + ab = a^2x^2 - a^2 - 4b^2x^2 + b^2$,

$2abx = a^2x^2 - a^2 - 4b^2x^2 + b^2$,

$4b^2x^2 - a^2x^2 + 2abx = b^2 - a^2$,

$(4b^2 - a^2)x^2 + 2abx = b^2 - a^2$.

Complete the square, multiplying by 4 times the coefficient of x^2 and adding the square of the coefficient of x,

$4(4b^2 - a^2)^2x^2 + (\ ) + (2ab)^2 = 16b^4 - 16b^2a^2 + 4a^4$,

$(4b^2 - a^2)2x + 2ab = \pm(4b^2 - 2a^2)$,

$(4b^2 - a^2)2x = 4b^2 - 2ab - 2a^2$,

or $2a^2 - 2ab - 4b^2$.

$$x = \frac{2b^2 - ab - a^2}{4b^2 - a^2}, \text{ or } \frac{a^2 - ab - 2b^2}{4b^2 - a^2}.$$

Ans. $x = \frac{b-a}{2b-a}$, or $-\frac{b+a}{2b+a}$.

8. Solve

$$x + y = 4 \quad (1)$$
$$x^4 + y^4 = 82 \quad (2)$$

Put $u + v$ for x, and $u - v$ for y.

(1) becomes $2u = 4$.

$\therefore u = 2$.

(2) becomes $u^4 + 16u^2v^2 + v^4 = 41$ (3)

Substitute 2 for u in (3), $16 + 24v^2 + v^4 = 41$,

$v^4 + 24v^2 = 25$,

$v^4 + (\ ) + 144 = 169$,

$v^2 + 12 = \pm 13$,

$v^2 = 1$ or -25,

$v = \pm 1$ or $\pm\sqrt{-25}$.

Ans. $\begin{cases} x = 3, 1, \text{ or } 2 \pm \sqrt{-25}, \\ y = 1, 3, \text{ or } 2 \mp \sqrt{-25}. \end{cases}$

9. Show that $2\sqrt[3]{a^3b^2}$, $\sqrt[3]{ab^3}$ $\frac{1}{2}\sqrt[3]{\frac{a^6}{b}}$ are similar surds.

$$2\sqrt[3]{a^3b^2} = 2a\sqrt[3]{b^2},$$
$$\sqrt[3]{ab^3} = 2b\sqrt[3]{b^2},$$
$$\tfrac{1}{2}\sqrt[3]{\frac{a^6}{b}} = \tfrac{1}{2}\sqrt[3]{\frac{a^6b^3}{b^4}} = \frac{a^2}{2b}\sqrt[3]{b^2}.$$

Since they all have the same surd factor they are similar surds.

10. Simplify

$$(2ab)^{\frac{1}{3}} \times (3ab^2)^{\frac{1}{3}} + (5ab^2)^{\frac{1}{3}}$$
$$= (2ab)^{\frac{1}{3}} \times (3ab^2)^{\frac{1}{3}} + (5ab^2)^{\frac{1}{3}}$$
$$= \sqrt[3]{(2ab)^2} \times \sqrt[3]{(3ab^2)^2} + \sqrt[3]{5ab^2}$$
$$= \sqrt[3]{2^2a^2b^2} \times \sqrt[3]{3^2a^2b^4} + \sqrt[3]{5ab^2}$$
$$= \sqrt[3]{2^2 \times 3^2 \times a^4 \times b^3} + \sqrt[3]{5ab^2}$$
$$= \sqrt[3]{\frac{2^2 \times 3^2a^3b^2}{5ab^2}}$$
$$= \sqrt[3]{\frac{2^2 \times 3^2 \times 5^2a^4b^4}{5^3}}$$
$$= \tfrac{1}{5}\sqrt[3]{225000a^4b^4}.$$

Ans. $\frac{1}{5}\sqrt[3]{225000a^4b^4}$.