

# **THE PHILOSOPHY OF MATHEMATICS**

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The philosophy of mathematics by Auguste Comte

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**AUGUSTE COMTE**

**THE PHILOSOPHY  
OF MATHEMATICS**



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THE  
PHILOSOPHY  
OF  
MATHEMATICS:

TRANSLATED FROM THE  
COURS DE PHILOSOPHIE POSITIVE  
OF  
AUGUSTE COMTE.

BY  
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## P R E F A C E.

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THE pleasure and profit which the translator has received from the great work here presented, have induced him to lay it before his fellow-teachers and students of Mathematics in a more accessible form than that in which it has hitherto appeared. The want of a comprehensive map of the wide region of mathematical science—a bird's-eye view of its leading features, and of the true bearings and relations of all its parts—is felt by every thoughtful student. He is like the visitor to a great city, who gets no just idea of its extent and situation till he has seen it from some commanding eminence. To have a panoramic view of the whole district—presenting at one glance all the parts in due co-ordination, and the darkest nooks clearly shown—is invaluable to either traveller or student. It is this which has been most perfectly accomplished for mathematical science by the author whose work is here presented.

Clearness and depth, comprehensiveness and precision, have never, perhaps, been so remarkably united as in AUGUSTE COMTE. He views his subject from an elevation which gives to each part of the complex whole its true position and value, while his telescopic glance loses none of the needful details, and not only itself pierces to the heart

of the matter, but converts its opaqueness into such transparent crystal, that other eyes are enabled to see as deeply into it as his own.

Any mathematician who peruses this volume will need no other justification of the high opinion here expressed; but others may appreciate the following endorsements of well-known authorities. *Mill*, in his "Logic," calls the work of M. Comte "by far the greatest yet produced on the Philosophy of the sciences;" and adds, "of this admirable work, one of the most admirable portions is that in which he may truly be said to have created the Philosophy of the higher Mathematics." *Morell*, in his "Speculative Philosophy of Europe," says, "The classification given of the sciences at large, and their regular order of development, is unquestionably a master-piece of scientific thinking, as simple as it is comprehensive;" and *Lewes*, in his "Biographical History of Philosophy," names Comte "the Bacon of the nineteenth century," and says, "I unhesitatingly record my conviction that this is the greatest work of our age."

The complete work of M. Comte—his "*Cours de Philosophie Positive*"—fills six large octavo volumes, of six or seven hundred pages each, two thirds of the first volume comprising the purely mathematical portion. The great bulk of the "Course" is the probable cause of the fewness of those to whom even this section of it is known. Its presentation in its present form is therefore felt by the translator to be a most useful contribution to mathematical progress in this country.

The comprehensiveness of the style of the author—grasping all possible forms of an idea in one Briarean sentence, armed at all points against leaving any opening for mistake or forgetfulness—occasionally verges upon cumbersomeness and formality. The translator has, therefore, sometimes taken the liberty of breaking up or condensing a long sentence, and omitting a few passages not absolutely necessary, or referring to the peculiar "Positive philosophy" of the author; but he has generally aimed at a conscientious fidelity to the original. It has often been difficult to retain its fine shades and subtle distinctions of meaning, and, at the same time, replace the peculiarly appropriate French idioms by corresponding English ones. The attempt, however, has always been made, though, when the best course has been at all doubtful, the language of the original has been followed as closely as possible, and, when necessary, smoothness and grace have been unhesitatingly sacrificed to the higher attributes of clearness and precision.

Some forms of expression may strike the reader as unusual, but they have been retained because they were characteristic, not of the mere language of the original, but of its spirit. When a great thinker has clothed his conceptions in phrases which are singular even in his own tongue, he who professes to translate him is bound faithfully to preserve such forms of speech, as far as is practicable; and this has been here done with respect to such peculiarities of expression as belong to the



author, not as a foreigner, but as an individual—not because he writes in French, but because he is Auguste Comte.

The young student of Mathematics should not attempt to read the whole of this volume at once, but should peruse each portion of it in connexion with the temporary subject of his special study: the first chapter of the first book, for example, while he is studying Algebra; the first chapter of the second book, when he has made some progress in Geometry; and so with the rest. Passages which are obscure at the first reading will brighten up at the second; and as his own studies cover a larger portion of the field of Mathematics, he will see more and more clearly their relations to one another, and to those which he is next to take up. For this end he is urgently recommended to obtain a perfect familiarity with the "Analytical Table of Contents," which maps out the whole subject, the grand divisions of which are also indicated in the Tabular View facing the title-page. Corresponding heads will be found in the body of the work, the principal divisions being in SMALL CAPITALS, and the subdivisions in *Italics*. For these details the translator alone is responsible.

## ANALYTICAL TABLE OF CONTENTS.

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### INTRODUCTION.

	Page
GENERAL CONSIDERATIONS ON MATHEMATICAL SCIENCE .....	17
THE OBJECT OF MATHEMATICS .....	18
Measuring Magnitudes .....	18
Difficulties .....	19
General Method .....	20
Illustrations .....	21
1. Falling Bodies .....	21
2. Inaccessible Distances .....	23
3. Astronomical Facts .....	24
TRUE DEFINITION OF MATHEMATICS .....	25
— A Science, not an Art .....	25
ITS TWO FUNDAMENTAL DIVISIONS .....	26
Their different Objects .....	27
Their different Natures .....	29
<i>Concrete Mathematics</i> .....	31
Geometry and Mechanics .....	32
<i>Abstract Mathematics</i> .....	33
The Calculus, or Analysis .....	33
EXTENT OF ITS FIELD .....	35
Its Universality .....	36
Its Limitations .....	37

BOOK I.  
ANALYSIS.

CHAPTER I.

	Page
GENERAL VIEW OF MATHEMATICAL ANALYSIS . . . . .	45
THE TRUE IDEA OF AN EQUATION . . . . .	46
Division of Functions into Abstract and Concrete . . . . .	47
Enumeration of Abstract Functions . . . . .	50
DIVISIONS OF THE CALCULUS . . . . .	53
<i>The Calculus of Values, or Arithmetic.</i> . . . .	57
Its Extent . . . . .	57
Its true Nature . . . . .	59
<i>The Calculus of Functions</i> . . . . .	61
Two Modes of obtaining Equations . . . . .	61
1. By the Relations between the given Quantities . . . . .	61
2. By the Relations between auxiliary Quantities . . . . .	61
Corresponding Divisions of the Calculus of Functions . . . . .	67

CHAPTER II.

ORDINARY ANALYSIS; OR ALGEBRA. . . . .	69
Its Object . . . . .	69
Classification of Equations . . . . .	70
ALGEBRAIC EQUATIONS . . . . .	71
Their Classification . . . . .	71
ALGEBRAIC RESOLUTION OF EQUATIONS . . . . .	72
Its Limits . . . . .	72
General Solution . . . . .	72
What we know in Algebra . . . . .	71
NUMERICAL RESOLUTION OF EQUATIONS . . . . .	75
Its limited Usefulness . . . . .	76
Different Divisions of the two Systems . . . . .	78
THE THEORY OF EQUATIONS . . . . .	79
THE METHOD OF INDETERMINATE COEFFICIENTS . . . . .	80
IMAGINARY QUANTITIES . . . . .	81
NEGATIVE QUANTITIES . . . . .	81
THE PRINCIPLE OF HOMOGENEITY . . . . .	84