

**A FIRST COURSE IN
PROJECTIVE
GEOMETRY**

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BY

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PREFACE

THIS work is intended for the use of students who have read the substance of Euclid, Books I.-XI., and who desire some introduction to the properties of the conic before proceeding to the study of the more advanced works on modern pure geometry.

The subject of Geometrical Conics, through the medium of which such an introduction is usually acquired, often proves repulsive to the average student.

In my opinion this is due to two causes: first, the demands which it makes upon the memory owing to its lack of coherence as commonly treated; and, secondly, to the very slight extension of outlook as regards method which it affords.

In the presentation here adopted, which, I venture to think, is in some respects original, I have endeavoured to overcome as far as possible these defects, while at the same time giving a slight sketch of the method of projection, and of the great principles of homography and duality upon which the further development of pure geometry so greatly depends.

No systematic treatment of imaginary elements or of the theory of involution is attempted, as these subjects are, in my judgment, unsuitable for a first course.

On the other hand, considerable use has been made of concrete illustrations in the form of examples involving practical drawing and calculation, as my teaching experience has con-

vinced me of their value in bringing home to the beginner the extent and variety of application of the principles involved.

With a view to further stimulating interest, a short historical note has been appended to most of the chapters.

In conclusion, I desire to express my hearty thanks to the Senate of the University of London for permission to make use of examples extracted from the degree examination papers of that University ; to my friend, Mr. W. H. Salmon, Lecturer in Mathematics at the Northampton Technical Institute, Clerkenwell, for his great kindness in reading the work both in manuscript and proof, and for his helpful criticisms and suggestions ; and to Mr. H. H. Smart for his valuable assistance with the photographs reproduced in the frontispiece.

E. HOWARD SMART.

ORPINGTON,

May 8th, 1913.

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