PROJECTIVE GEOMETRY, VOLUME I

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Projective Geometry, Volume I by Oswald Veblen & John Wesley Young

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PREFACE

Geometry, which had been for centuries the most perfect example of a deductive science, during the creative period of the nineteenth century outgrew its old logical forms. The most recent period has however brought a clearer understanding of the logical foundations of mathematics and thus has made it possible for the exposition of geometry to resume the purely deductive form. But the treatment in the books which have hitherto appeared makes the work of laying the foundations seem so formidable as either to require for itself a separate treatise, or to be passed over without attention to more than the outlines. This is partly due to the fact that in giving the complete foundation for ordinary real or complex geometry, it is necessary to make a study of linear order and continuity, — a study which is not only extremely delicate, but whose methods are those of the theory of functions of a real variable rather than of elementary geometry.

The present work, which is to consist of two volumes and is intended to be available as a text in courses offered in American universities to upper-class and graduate students, seeks to avoid this difficulty by deferring the study of order and continuity to the second volume. The more elementary part of the subject rests on a very simple set of assumptions which characterize what may be called "general projective geometry." It will be found that the theorems selected on this basis of logical simplicity are also elementary in the sense of being easily comprehended and often used.

Even the limited space devoted in this volume to the foundations may seem a drawback from the pedagogical point of view of some mathematicians. To this we can only reply that, in our opinion, an adequate knowledge of geometry cannot be obtained without attention to the foundations. We believe, moreover, that the abstract treatment is peculiarly desirable in projective geometry, because it is through the latter that the other geometric disciplines are most readily coördinated. Since it is more natural to derive the geometrical disciplines associated with the names of Euclid, Descartes, Lobatchewsky, etc., from projective geometry than it is to derive projective geometry from one of them, it is natural to take the foundations of projective geometry as the foundations of all geometry.

The deferring of linear order and continuity to the second volume has necessitated the deferring of the discussion of the metric geometries characterized by certain subgroups of the general projective group. Such elementary applications as the metric properties of conics will therefore be found in the second volume. This will be a disadvantage if the present volume is to be used for a short course in which it is desired to include metric applications. But the arrangement of the material will make it possible, when the second volume is ready, to pass directly from Chapter VIII of the first volume to the study of order relations (which may themselves be passed over without detailed discussion, if this is thought desirable), and thence to the development of Euclidean metric geometry. We think that much is to be gained pedagogically as well as scientifically by maintaining the sharp distinction between the projective and the metric.

The introduction of analytic methods on a purely synthetic basis in Chapter VI brings clearly to light the generality of the set of assumptions used in this volume. What we call "general projective geometry" is, analytically, the geometry associated with a general number field. All the theorems of this volume are valid, not alone in the ordinary real and the ordinary complex projective spaces, but also in the ordinary rational space and in the finite spaces. The bearing of this general theory once fully comprehended by the student, it is hoped that he will gain a vivid conception of the organic unity of mathematics, which recent developments of postulational methods have so greatly emphasized.

The form of exposition throughout the book has been conditioned by the purpose of keeping to the fore such general ideas as group, configuration, linear dependence, the correspondence between and the logical interchangeability of analytic and synthetic methods, etc. Between two methods of treatment we have chosen the more conventional in all cases where a new method did not seem to have unquestionable advantages. We have tried also to avoid in general the introduction of new terminology. The use of the word *on* in connection with duality was suggested by Professor Frank Morley.

We have included among the exercises many theorems which in a larger treatise would naturally have formed part of the text. The more important and difficult of these have been accompanied by references to other textbooks and to journals, which it is hoped will introduce the student to the literature in a natural way. There has been no systematic effort, however, to trace theorems to their original sources, so that the book may be justly criticized for not always giving due credit to geometers whose results have been used.

Our cordial thanks are due to several of our colleagues and students who have given us help and suggestions. Dr. H. H. Mitchell has made all the drawings. The proof sheets have been read in whole or in part by Professors Birkhoff, Eisenhart, and Wedderburn, of Princeton University, and by Dr. R. L. Börger of the University of Illinois. Finally, we desire to express to Ginn and Company our sincere appreciation of the courtesies extended to us.

O. VEBLEN
J. W. YOUNG

August, 1910

In the second impression we have corrected a number of typographical and other errors. We have also added (p. 343) two pages of "Notes and Corrections" dealing with inaccuracies or obscurities which could not be readily dealt with in the text. We wish to express our cordial thanks to those readers who have kindly called our attention to errors and ambiguities.

O.V.
J.W.Y.

August, 1916

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