

MANUAL OF PLANE TRIGONOMETRY

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Manual of Plane Trigonometry by James Henchie

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JAMES HENCHIE

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TRIGONOMETRY**

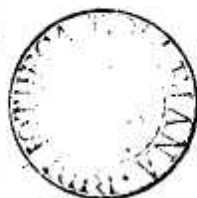
MANUAL
OF
PLANE TRIGONOMETRY.

BY

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PREFACE.

My chief object in writing the present work is to lay before the student in a plain and compact form all that is necessary in the subject towards helping him to obtain a First Class Pass (Stages II., III) in the May Examinations conducted by the Science and Art Department, South Kensington. Having had several years' experience in teaching the subject, I found, as a rule, that students required a small work dealing only with what was absolutely necessary for their examination, as the ordinary standard Treatises on Trigonometry were, on the one hand, too expensive for them, and, on the other, dealt with matter far beyond their requirements. This volume, it is to be hoped, will clear away those difficulties. I have striven to make the matter clear and intelligible, and how far I have succeeded remains to be seen.

At the end of each chapter I have given the solutions in full of a number of problems that were given at the May Examinations during the last four or five years, besides giving hints on others; so that the student may make himself familiar with the manner and principle on which such questions are solved.

The examples have been carefully graduated, and it is to be hoped the answers will be found correct. The Author will feel obliged for any corrections made through the Publisher.

J. HENCHIE.

UNITED WESTMINSTER SCHOOLS.

January 1st, 1877.

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We can express the minutes and seconds in the centesimal method, as the decimal fraction of a grade, very conveniently, since a minute is the hundredth part of a grade, and a second the hundredth part of a minute. Thus the above might be written $12^{\circ}1575$.

4. The two methods might be compared, and we can thus express degrees in terms of grades, and *vice versa*.

Let D = number of degrees in any angle,
and G = number of grades in same angle ;

then we have the proportion—

$$D : G :: 90 : 100$$

$$\therefore D = \frac{9}{10} G = G - \frac{1}{10} G \dots\dots (a)$$

Also from the same proportion—

$$G = \frac{10}{9} D = D + \frac{1}{9} D.$$

Thus we get the following rules :—

(a.) *To reduce grades to degrees, we must subtract one-tenth of the number of grades from the whole, and the remainder gives us the number of degrees.*

(β .) *To reduce degrees to grades, we must add one-ninth of the number of degrees to the whole, and the sum gives us the number of grades.*

We give an example of each :—

Express $85^{\circ} 5' 45''$ in degrees.

Now $85^{\circ} 5' 45'' = 85^{\circ} \cdot 0545$.

$$\therefore 85 \cdot 05450$$

Sub. $\frac{1}{10}$ th of this = $8 \cdot 50545$

Number of degrees = $76 \cdot 54905$

$$\begin{array}{r} 60 \\ 32 \cdot 94300 \\ 60 \\ \hline 56 \cdot 580 \end{array} \quad \text{Ans. } 76^{\circ} 32' 56'' \cdot 58$$

Express $76^{\circ} 32' 56'' \cdot 58$ in grades.

$$\begin{array}{r} 60 \overline{) 56 \cdot 58} \\ \underline{60 \overline{) 32 \cdot 943}} \\ 76 \cdot 54905 \end{array}$$

$$\text{Add } \frac{1}{4} \text{th of this} = 8 \cdot 50545$$

$$\text{Number of grades} = \frac{85 \cdot 05450}{1} = 85^{\circ} 5' 45''$$

5. We might also express English minutes in terms of French minutes, and English seconds in terms of French seconds.

Let M = number of English seconds in any angle, and m = number of French seconds in same angle.

$$\therefore M : m :: 90 \times 60 : 100 \times 100$$

$$\therefore M = \frac{90 \times 60}{100 \times 100} m = \frac{27}{50} m.$$

$$\text{Also } m = \frac{50}{27} M.$$

Again, if S = number of English seconds in any angle, and s = number of French seconds in same angle, it may be found that

$$S = \frac{81}{250} s.$$

$$\text{And } s = \frac{250}{81} S.$$

EXERCISES.

- Express $38^{\circ} 6' 39'' \cdot 74$ in grades, &c.
- Express $65^{\circ} 15' 20''$, $30^{\circ} 25' 35''$, $48^{\circ} 30' 30'' \cdot 75$ in grades.
- Find how many degrees, minutes, and seconds in each of the following:—

$$66\frac{1}{2}^{\circ}, 10^{\circ}, 90^{\circ} 75' 25'' \cdot 75.$$

4. How many sides has a polygon which contains as many grades in all its angles as there are degrees in the angles of a twelve-sided figure?

5. The number of sides of two regular polygons are in the ratio of 9 : 6, and the number of degrees in an angle of the one is equal

to the number of grades in an angle of the other; find the number of sides in each polygon.

N.B.—In any polygon, if n = number of sides, then (Euc. I. 32 Cor.) $2n - 4$ = number of right angles in all its angles \therefore

$$\frac{2n - 4}{n} \times 90 = \text{number of degrees in each angle.}$$

6. What is the length of an arc in inches, which subtends an angle of $1'$ at a distance of 100 yards?

7. If the path described by the earth round the sun be supposed circular, find its velocity if its distance from the sun be 95,000,000 miles.

N.B.—Chapter II. must be read before this can be attempted.

8. If one-sixth of a right angle or 15° be taken as the unit of angular measurement, by what number would an angle of 60° be represented?

9. Divide $96\frac{3}{4}^\circ$ into two parts, so that the number of grades in one part shall bear to the number of degrees in the other part the ratio of 9 : 5.

10. The sum of two angles is 60° and their difference is 18° ; find them.

11. If the unit of angular measurement be x° , find an expression for an angle of y° .

12. What is meant by the centesimal and sexagesimal divisions of an angle?

13. If the radius of a circle be 97 ft., and the angle at the centre $73\frac{3}{4}$, find the length of the arc subtending the angle.

14. One angle of a triangle contains as many degrees as another contains grades, and the third angle is half the sum of the other two; find the number of degrees in each angle.

15. The angle which is subtended by an arc equal in length to the radius is $2062648''$. Reduce this to grades and decimals, and thence deduce the value of $\frac{1}{\pi}$

16. Reduce the following to degrees, &c. :—

$$\begin{array}{l} 20^\circ 33\frac{1}{2}' 40'' 50''' 70'''' 95'''''' \\ 25^\circ 44' 89'' \end{array}$$

17. Reduce to grades, &c. :—

$$\begin{array}{l} (a) 31^\circ 45' 57'' \\ (b) 11^\circ 15' \\ (c) 22^\circ 30' \\ (d) 75^\circ \\ (e) 78^\circ 45' \end{array}$$