TABLE OF QUARTER-SQUARES OF ALL INTEGER NUMBERS UP TO 100,000, BY WHICH THE PRODUCT OF TWO FACTORS MAY BE FOUND BY THE AID OF ADDITION AND SUBTRACTION ALONE

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BY

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PREFACE.

In December last the author had the honor of reading before the Institute of Actuaries, a Paper "On a Method of finding the Product of two Factors by means of the Addition and Subtraction of Natural Numbers,"* in which he gave some account of the construction and use of the Table forming the subject of the present work. The Paper referred to was based upon the suggestions contained in an article contributed to the Journal of the Institute, entitled, "On Multiplication by a Table of Single Entry," by J. J. Sylvester, Esq., M.A., F.R.S.,† in which he shows that, by means of a Table of Squares, the product of any two numbers may be found "by the aid of the processes of addition and subtraction alone, just as is the case in logarithmic computation, but with the advantage over that method of perfect precision in the result, and of the number of findings for each computation being only two in number, the third, and by far the most troublesome of the three findings, viz., that of finding back the number corresponding to the result of the linear (meaning thereby additive or subtractive) process, which occurs in the logarithmic, not arising in the method about to be explained."

The present work is offered to the public at the suggestion of several eminent actuaries who have expressed a favorable opinion of its utility; and it is with the impression that it will be of considerable service to computers generally, that the author has been encouraged to undertake the superintendence of the publication of so great a mass of figures; and should it answer the end in view, he will be amply repaid for the trouble it has cost him.

It may be proper to remark, that at the time of the author's reading the paper before referred to, and until after he had made known his intention to publish, he was not aware of the existence of any similar Table; and it was only through the courtesy of Mr. Peter Gray,—so favorably known as the author of several valuable contributions

* *Assurance Magazine*, vol. vi., p. 121. † *Assurance Magazine*, vol. iv., p. 236.

to the science of Life Assurance—that his attention was drawn to the works of VOISIN and MERFAUT,* and the article "Quarter-Squares," in the *Penny Cyclopædia*, as well as to an article by Professor SYLVESTER, in the *London and Edinburgh Philosophical Magazine*, vol. vii., p. 430. It is also due to Mr. GRAY to mention, that he has in his possession the MS. of a Table of Quarter-Squares by Major SHORTREDE, extending to 200,000. Had the author been sooner made aware of the existence of Major SHORTREDE's Table, he would not willingly have encountered the labour of computing, and afterwards carrying through the press, the table contained in this volume.

The article "Quarter-Squares," in the *Penny Cyclopædia*, mentions that a Table of the fourth part of the squares of numbers may be substituted for one of logarithms in multiplication. For since $\frac{(a+b)^2}{4} - \frac{(a-b)^2}{4} = ab$, a table which gives the squares of the halves of numbers will, by the subtraction† of the squares of the halves, or quarter-squares, give the product; and it is added, that such a table had been published in France, and was introduced to a certain extent in the late Professor LESLIE's *Philosophy of Arithmetic*.

Turning to the latter work, the following remarks occur in the Preface (2nd Edit., 1820): "The most valuable addition I have made, consists in the Table of Quarter-Squares, near the end of this volume, which, to a certain extent, perform the multiplication of numbers more expeditiously than even logarithms themselves." And at p. 257, it is added, "This application of a Table of Quarter-Squares, as it is derived from the simplest principles, might have readily occurred to a mathematician; yet I have no where seen it brought into practical use till last summer, I met with, at Paris, a small book by ANTOINE VOISIN, printed in 1817." LESLIE then gives a Table of Quarter-Squares, from 1 to 2000, as a specimen, and adds: "It would be a great service, however, in facilitating many calculations, to have the whole Table reprinted, or perhaps even extended to 200,000, which might be condensed into a moderate sized volume."

The work of VOISIN is entitled, *Tables de Multiplications ou Logarithmes des Nombres Entiers depuis* 1 *jusqu'a* 20,000. In his preface, after mentioning the advantages to calculators afforded by logarithms, he remarks, "L'admirable calcul de logarithmes indique, il est vrai, les moyens de trouver à quels nombres correspondent ceux qui surpassent les limites des tables, mais ces abréviations ne diminuent pas sensiblement

* The author is also indebted to Mr. E. SANG, for referring him to VOISIN's Tables.

† In the original article, the word *addition* was used, evidently by an oversight on the part of the author.

le travail; aussi pour ces cas la méthode ordinaire a-t-elle prévalu. Les tables que nous offrons au public, en abrégeant considérablement les calculs, ont l'avantage de donner directement le produit de deux nombres, de servir de preuves aux opérations faites par les logarithmes ordinaires, et enfin d'indiquer les moyens de corriger les fautes qui auraient pu échapper aux lectures des épreuves." And in his concluding remarks, VOISIN proposed to publish a second volume, containing the "*logarithms*" of numbers (for so he is pleased to call them,) from 20,000 to 100,000; but whether this intention was ever carried into practice, the author has not been able to ascertain. It may be remarked, that the arrangement adopted by VOISIN is faulty, owing to the figures not being grouped, whereby it is rendered difficult accurately to set down numbers under those previously extracted, in the course of an operation; as also, being in single vertical columns, the whole of the leading figures which remain constant for many successive numbers, are necessarily repeated, and which would not have been required in the more convenient and compact form in which it is here presented.

Another work having the same object as the last, is entitled *Tables Arithmonomiques fondées sur la rapport du rectangle au carré, ou Le Calcul réduit à son dernier degré de simplification*, par J. M. MERPAUT, Professeur de Mathematiques Speciales au Collége de Vannes (à Vannes, 1832), and is designed to facilitate the processes of multiplication and division, by means of a Table of Quarter-Squares, from 1 to 40,000, and of a Table of Reciprocals, from 1 to 10,000. The Tables in this work are an improvement upon VOISIN's, for the digits of each quarter-square and reciprocal are grouped in periods of three, as shown in the following example of the first and last lines of p. 228 of that Table:—

22,8

	0	1	2	3	4	5	6	7	8	9
0	129 960 000	130 074 025	130 188 100	130 302 225	130 416 400	130 530 625	130 644 900	130 759 225	130 873 600	130 988 025
*	*	*	*	*	*	*	*	*	*	*
*	*	*	*	*	*	*	*	*	*	*
9	130 062 620	130 176 690	130 290 810	130 404 980	130 519 200	130 633 470	130 747 790	130 862 260	130 976 580	131 091 050

From which it will be seen that the first three figures of the argument are to be sought at the head of the Table, the fourth figure at the head of one of the vertical

columns, in which, and even with the final figure in the column at the left hand will be found the quarter-square required; thus, the quarter-square of 22890 is 130988025, and the quarter-square of 22859 is 130633470. This arrangement, although peculiar and somewhat inconvenient, is superior to that of VOISIN's, but is open to the objection of occupying much space, and does not save the repetition of the constant figures occurring at the beginning of many successive quarter-squares. In addition to the method of using the Table as explained by VOISIN, the author has also shown the mode of obtaining the quarter-squares of numbers, and the products of numbers, beyond the limits of the Table, by the aid of numbers therein tabulated, which adds materially to its utility in such cases.

The only other work known to the present author, in which a Table of Quarter-Squares is inserted or referred to, is GALBRAITH'S *General Tables*, 2nd Edit., 1836, intended as a supplement to the second edition of his *Mathematical and Astronomical Tables*, in which, at p. 17, the following observations are made: "Table XXXIV* will be found very useful for obtaining the products of numbers in general, and of those employed in the preceding [astronomical] calculations in particular. In many instances, calculations may be performed more readily by this table than by logarithms. . . . . Besides, in using this Table, the differences of the sums set down are the results required, whereas the corresponding natural numbers must first be found from a Table of Logarithms, in searching for which a considerable time is often necessary." The author also observes, "that he has given to the Table double arguments both in natural and sexagesimal numbers, which he hoped would render its utility still more extensive than that given originally by VOISIN;" and adds, that "it will also be found, in conjunction with a Traverse Table, to be highly useful in surveying with a compass, as practised in uncleared countries, or newly discovered Islands circumnavigated by a boat, or any small decked vessel, whose courses and distances are carefully kept, and the area in square miles is required tolerably near the truth." And finally, he draws attention to the utility of the Table for facilitating the calculations of surveyors.

It is not a little remarkable, that LESLIE and GALBRAITH both repeat an error which occurs in VOISIN's Table, thus showing beyond doubt, that they merely reprinted from VOISIN. The error referred to is in the quarter-square of 747, which in all three works appears as 139052, the correct number being 139502.

* Quarter Squares of Numbers from 1 to 3149.

The author, notwithstanding he has not the merit of being the first to compute a Table of Quarter-Squares, yet hopes that he will have rendered a service to computers, for at least he will have presented a method, not hitherto capable of being extensively adopted, owing to the works referred to being but little known, and which, if not absolutely scarce, cannot easily be procured in this country. The great extension of the Table as given in this volume, adds also to its superiority over the others, giving it a power ten times greater than VOISIN'S, and five times greater than that of MERFAUT'S, and supplying the means of operating directly upon all factors of five digits.

While the value of Logarithms is admitted to the fullest extent, yet there are cases in which a Table of Quarter-Squares has advantages over them. With logarithms, attention is necessary to their characteristic or index, and its distinction into positive or negative, according as the natural numbers are greater or less than unity; and there is still further the third reverse finding to obtain the number answering to the logarithmic result. This very much increases the labour of the computer, and requires his constant attention, and may in some degree account for the fact, that logarithms are not more generally used in ordinary calculations. But with a Table of Quarter-Squares we have to do with none of these considerations, for the difference of the Quarter-Squares of the sum, and difference of numbers to be multiplied, gives directly their product without any further tabular entry. The advantage of the Table over a Table of Logarithms, will be apparent in many cases of mensuration, *e.g.*, in finding the Areas of Polygons, some of which cannot be conveniently found by logarithms.

In conclusion, the author has taken considerable pains to render the Table as accurate as possible. Having been entirely calculated in duplicate, the types were set up from one of the workings, and the proofs read against the duplicate. After correction, the several pages were stereotyped, and proofs from the plates again read against the duplicate; defective figures were then marked and corrected; and finally, the sheets were printed from the stereotype plates. From the careful revision the Table has thus undergone, it is believed that it will be found as free from errors of every kind as any similar work of equal extent.

The Table is therefore submitted to the public in the belief that it will be found useful both in the study of the practical computer and in the counting house of the merchant; and will facilitate, by the use of decimal arithmetic, a vast number of calculations occurring in commerce as well as in the sciences. To the banker in computing interest on accounts current, &c.; to the architect and civil engineer in estimating