ALGEBRA SELF-TAUGHT

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649030286

Algebra Self-Taught by W. P. Higgs

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

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'SCIENTIFIC NOTES FOR UNSCIENTIFIC PEOPLE,' ETC.

LONDON:
E. & F. N. SPON, 48, CHARING CROSS.
NEW YORK:
446, BROOME STREET.
1876.

INTRODUCTORY.

Numerous attempts have been made to interest the general reader in the science of mathematics. Some of these attempts have succeeded; others, and the major portion, have failed. A careful study of the cause of failure leads to the observation of a striking similarity in the unsuccessful attempts; and this similarity lies in dealing to excess with abstract ideas. Mathematics is a tool invaluable when applied to the shaping of some end; but it is possible, even upon grinding, sharpening, or adjusting a tool, to waste time. Indeed, in the cutting of rough wood an edge may be too keen. So it is with mathematics; and the reader who wishes to progress may be made to feel that he is constantly at the grindstone. He may admit that by pre-eminence mathematics is the science of abstraction; but experience has taught him that the greatest good has arisen from its application to concrete methods.

Nor would it be difficult to show that mathematicians, by an excessive devotion to the science in its purity (the proper term for its fullest abstraction), have rendered it unpopular. In its transcendental dress the student approaches mathematics with awe; and, if bold enough to question, receives its teachings, as Professor Airy has aptly said, "rather with the doubts of imperfect faith than with the confidence of rational conviction." In his 'Principles of the Differential Calculus,' Mr. Woolhouse observes, with only too much reason, how "it is to be regretted that most of our academical treatises on this, as well as other subjects, abound so much with complex algebraic processes, without the slightest traces of logical reasoning to exercise and improve the intellect. We should bear in mind that the simple execution of analytical operations acquired by dint of practice and experience is a mere common species of labour, often merely mechanical; whilst a distinct apprehension of the specific object and meaning of the operations, and a contemplation of the clearness and beauty of the various arguments employed, constitute the intellectual lore that gratifies and enriches the mind, and stimulates its energies with an ardour after the investigation of truth."

Mathematics naturally divide into two divisions, to which are related (α) the solutions of problems where it is the object to ascertain the value of unknown (from the relation which they bear to known) and constant quantities; and (β) the investigation of variable quantities increasing or decreasing uniformly with regard to other quantities that are constant. When dealing with (α) we are in the dominions of ordinary *Algebra;* the consideration of β is effected by means of the Differential and the Integral Calculus. In the present work, then, we have only to take into view *constant* quantities, or quantities to which, if a value is once assigned, retain that value to the end of the problem.

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CHAPTER I.

SYMBOLS AND THE SIGNS OF OPERATION.

IN arithmetic we have learned to consider two kinds of numbers, *abstract* and *concrete*. 1, 2, 3, or 4, etc., is an abstract number, because it denotes an abstract idea of how many times; but 1 apple, 2 apples, or 3 apples, etc., is a concrete number, because it denotes so many actual substances. We cannot see or feel 2 as a number until it is embodied in the form of the two apples.

In algebra, however, the division into concrete and abstract need not be a source of trouble; for we may substitute a *letter* (a, b, c . . . . . x, y, z) for either an abstract or concrete number. Thus a may be put to represent 1 or 1 apple, or 2 or 2 apples. If a be taken as a kind of short way of writing one apple, $2a$ will stand for two apples; but if a is taken to represent two apples, then $2a$ will stand for four apples; and so on. Similarly, if a be taken to represent the figure 1, then $2a$ will be equal to 2. If a stand for 2, then $2a$ will equal 4.

This is enough, perhaps, to enable the student to understand the definition—*Algebra is the Science of Computation by Symbols.* The symbols generally employed are the letters of the alphabet, sometimes those of the Greek alphabet. There is no reason why any system of symbols, however arbitrary, should not have been used; but universal custom has given the preference to letters, arbitrary symbols being employed when the number of things or of abstract ideas to be represented exceeds the number of letters.

It will be convenient to remark that there is in common use in algebra a sign for equality, or for the words *equal to*. It is $=$. If we wish to express that one and one make or equal two, we can write 1 and $1 = 2$.

That also there is a sign for addition, called *plus*, and it is $+$. So that we can state shortly 1 added to $1 = 2$, or $1 + 1 = 2$. Again, there are signs for subtraction, multiplication, and division, to which we must presently refer.

There is some difference between the operations of arithmetic and those of algebra. Instance may be taken in adding 2 and 3. Arithmetically $2 + 3 = 5$; but if we place a for 2 and b for 3, we can only give the result in algebra as $a + b$, unless we previously arrange that c shall equal 5. In the last case we can say $a + b = c$, one of the simplest forms of an algebraic equation. Let us quite understand this. If we have a basket of fruit containing apples (call them a), and blackberries (call them b); we cannot say *apples* + *blackberries* equal *pears*; we can only give the result as *fruit*, which is a general and not a particular term. Consequently we must be satisfied with the statement of the contents of the basket (if we wish to describe particularly) as being apples and blackberries. Similarly, in algebra, we often cannot describe a more definite result than $a + b$.

What does this negative evidence teach? It shows that the sign $+$ is a sign indicating that the action of addition is to be performed—is, in fact, a sign of operation. So are all the signs in algebra. They indicate *what is to be done* when we have found the values of the symbols or letters which they connect. A complete formula, indeed, is nothing more than a *symbolic* direction to perform certain actions, to attain a desired result, which result may or may not be expressed.

CHAPTER II.

THE EQUATION AND THE UNKNOWN QUANTITY.

We have shown how the addition of 3 to 2 and the sum obtained may be shortly written, viz. $3 + 2 = 5$. This (again taking a for 2, b for 3, and c for 5), it has also been shown, may be represented algebraically as

$$a + b = c.$$

From the *sign of equality* (=) connecting the two members, all similar equal quantities so connected are classed under the general term of an *equation*. The *solution of an equation*, or the *solving* of an equation, merely means the working of it out until the *answer* has been found. The *answer* is the *unknown quantity*.

The unknown quantity is generally represented by the letter x, or, if there are more than one, by the letters x, y, z (known quantities, as we have said, being represented by the letters a, b, c . . . . . . etc.).

Substituting, therefore, the letter x for the letter c in the foregoing equation, we have

$$a + b = x \text{ (or the answer).}$$

As we have determined that a shall equal 2 and b shall equal 3, x, or the unknown quantity, or the answer, must equal 5. Thus we have as synonymous terms

$$a + b = x,$$

or

$$2 + 3 = 5.$$

Now we may so ring the changes on the two members of the equation as to form *new* equations. We say *new*, because for *two equations to be equal (or to be equated) their results (or their x's) must be equal.*

To test this, let us add 2 to *both* sides of the equation. We have

$$2 + 3 + 2 = 5 + 2, \text{ or } 7.$$

Evidently a new equation, because 5, the former result (or x), does not equal 7. Similarly, if we multiply both sides by 2, or divide them by 2, or take 2 from them, we shall obtain new equations. We learn, then, that the balance of an equation is not destroyed when we treat in any way *both* sides by the same quantity. This is the characteristic of an equation, and from this characteristic there springs a valuable aid to the solution of (or finding the answer to) equations.

Let us try to understand its application.

There is a sign employed in algebra (and sometimes in arithmetic) indicating *subtraction* or *taking from:* the sign is called *minus*, and is $-$. So that we may state the equation, $3 - 2 = 1$, or $b - a = x$. Suppose we are told (adhering to our former values for a and b) that

$$a = x - b,*$$

* The student will understand that I am dealing with these simple quantities in order that he may not have to think of the actions of his mind; for he will presently perceive that what is true of, and can be done with, these simple quantities, is as equally true of others, however complex.