

**THE PRACTICE AND THEORY OF ARITHMETIC,
CONTAINING DEFINITIONS OF TERMS, AND
RULES OF OPERATIONS, WITH CRAMPLES OF
METHODS, AND THE PRINCIPLES EXPLAINED IN
PROPOSITIONS, THE WHOLE FORMING A
COMPLETE TREATISE ON PRACTICAL AND
THEORETICAL ARITHMETIC**

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The Practice and Theory of Arithmetic, Containing Definitions of Terms, and Rules of Operations, with Cramples of Methods, and the Principles Explained in Propositions, the Whole Forming a Complete Treatise on Practical and Theoretical Arithmetic by W. F. Greenfield

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W. F. GREENFIELD

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PRACTICAL AND THEORETICAL ARITHMETIC.

BY THE
REV. W. F. GREENFIELD, M.A.
LATE SCHOLAR OF PEMBROKE COLLEGE, CAMBRIDGE,
SECOND AND MATHEMATICAL MASTER IN QUEEN ELIZABETH'S SCHOOL, IPSWICH.

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PREFACE.

The writer of the present work believes that all persons will agree in allowing that the ends, which ought to be kept in view in teaching any branch of Science, are these:—1st, That Principles be well understood; 2nd, That Definitions of terms be known; 3rd, That Rules for the application of the Principles to Practice be learnt; 4th, That neat methods of working be adopted; 5th, That a facility in expressing by writing the Principles, and the reasoning by which the Rules are established, be acquired. The *first* is necessary, because no Science can be said to be *taught*, unless a knowledge and comprehension of Principles be secured: the *second* is necessary, because, if definitions are not known, all reasoning on the subject will be as it were expressed in an unknown language: the *third* is required, because the main object of all Science is *Practical utility*, and it cannot be expected that on all occasions printed Rules can be before the eye: the *fourth* is desirable, if not necessary, in the same degree as a neat method of writing is so: the *fifth* will appear an object of great importance, if it be considered that the best method of testing a person's knowledge of a subject is by requiring him to write his ideas upon it; if he understand it, he will have no difficulty in expressing himself correctly; but if he do not, he will certainly shew the fact by his defective reasoning.

Now Definitions and Rules can only be *learnt* accurately from books, because they require to be expressed in precise terms in order to fix themselves in the memory; neat methods of working too must be shewn to the eye; but Principles cannot be learnt thoroughly from a book, without *oral* explanations, those contained in a book being necessarily concise, and such as themselves require elucidation and illustration: on the other hand, it is found that Principles may very well be taught, by a *competent* teacher, without a book, by repeated oral explanations, by illustrations, and by continual interrogations put to the learner in every variety of form. And that this is by far the better method of teaching will appear on these grounds, because it may be thereby ensured that no advance is made from one step to another without every previous step being understood, and because a fuller explanation is made compulsory on the part of the teacher, than he would be inclined to give, if any part could be supplied by a book, and because young pupils are *very apt* to overlook all the explanatory

portion of a book, being anxious for the most part to come as quickly, as may be, to the actual work of the subject, which is always more attractive, and generally *pays best* in a school. For these reasons it appears that oral teaching is preferable; nor will it occupy any longer time of the teacher, for teaching by book must be accompanied by oral teaching to be of *any use*, and this must be full and complete to be effective; if it be not full, the teacher will find that he will have to do his work over and over again, and that his pupils will make but slow and imperfect progress. And further, in a large school where many are taught in a class, it frequently happens that it is *assumed* that *all* understand a Principle, which *one* can explain according to a book, which all have given them to study, whereas the assumption ought certainly to be, that no one understands anything at all about it.

From what has preceded, it will have been seen that, in the opinion of the writer, the best mode of teaching any science in a school is by orally explaining, illustrating, and questioning upon, the principles; by supplying definitions and rules to be learnt by heart; by placing before the pupils neat methods of setting down their work; and by requiring them to explain the reasons of the processes *vivâ voce* and by writing.

The first portion of the present work contains the Rules and Definitions of Arithmetic, with examples of processes worked out in full, and in the precise form in which they ought to be shewn to a Master. The second portion of the work contains the Principles, exhibited in *Propositions*, as specimens of the form in which the pupil should give written explanations of processes. The former of these portions is all that is required in elementary classes, the second will furnish a useful exercise for more advanced pupils, and will serve as a good introduction to the "book-work" of Algebra, and containing, as it does, a gradual and complete explanation of principles, will be found to be an important assistant both to the student and to the teacher.

The writer has found, that in most works on Arithmetic the Rules and Definitions are spread over many pages of explanation, and in many instances not distinctly or connectedly given, so that the pupil has to spend time in seeking for them, and in connecting them. He has therefore exhibited every Rule and Definition distinctly and connectedly, in such a form as may be easily committed to memory; and it is hoped that the two portions of the work will be found to contain all that is necessary for imparting an accurate knowledge of both the Practice and Principles of Arithmetic in the hands of a competent instructor.

It is intended, if circumstances permit, to publish, on the same plan, The Practice and Theory of Algebra.

February 25, 1853.

W. F. G.

CONTENTS.

PART I.

I.—NOTATION AND NUMERATION.

	PAGE
Definitions - - - - -	1
Numeration table - - - - -	3
To write in figure a given number - - - - -	3
To read the number represented by a series of figures - - - - -	3

II.—SIMPLE ADDITION.

To add several numbers together - - - - -	4
---	---

III.—SIMPLE SUBTRACTION.

To subtract one number from another - - - - -	4
---	---

IV.—SIMPLE MULTIPLICATION.

To multiply together two numbers not greater than twelve - - - - -	5
To multiply a number greater than twelve, by one not greater than twelve - - - - -	5
To multiply by 10, 100, 1000, &c. - - - - -	6
To multiply by any number greater than twelve - - - - -	6

V.—SIMPLE DIVISION.

To divide one number by another - - - - -	7
Questions in the four simple rules - - - - -	9

VI.—MEASURES.

Definitions - - - - -	9
To resolve a number into its prime factors - - - - -	10
To find the greatest common measure of two or more numbers - - - - -	10

VII.—MULTIPLES.

Definitions - - - - -	12
To find the least common multiple of two or more numbers - - - - -	12

VIII.—FRACTIONS.

	PAGE
Definitions	12
A.—REDUCTION OF FRACTIONS.	13
To reduce a fraction to its lowest terms	13
To reduce an improper fraction to a mixed number, or an integer	14
To reduce an integer to an improper fraction with a given denominator	14
To reduce fractions to their least common denominator	15
B.—ADDITION OF FRACTIONS	15
To add fractions	15
To reduce a mixed number to an improper fraction	16
C.—SUBTRACTION OF FRACTIONS	17
To subtract one simple fraction from another	17
To subtract one mixed number from another	17
To subtract several fractions from the sum of several others	18
D.—MULTIPLICATION OF A FRACTION BY AN INTEGER	19
E.—DIVISION OF A FRACTION BY AN INTEGER	20
F.—REDUCTION OF A COMPOUND FRACTION TO A SIMPLE FRACTION	20
G.—MULTIPLICATION OF FRACTIONS	21
To multiply fractions together	21
To multiply a simple or compound fractional expression by a simple fraction, or by another compound expression	22
H.—DIVISION OF FRACTIONS	22
To divide one fraction by another	22
To divide a simple or compound fractional expression by a simple fraction, or by a compound expression	23
I.—REDUCTION OF COMPLEX FRACTIONS	24
K.—SIMPLIFICATION OF FRACTIONAL EXPRESSIONS	26

IX.—DECIMAL FRACTIONS.

To write a fraction with ten, or any power of ten for its denominator, as a decimal fraction	26
To convert a decimal into a vulgar fraction, or the sum of several fractions	29
To add decimals	29
To subtract decimals	29
To multiply decimals	30
Abbreviated method of multiplication of decimals, when only a certain number of decimal places are required in the product	30
Division of decimals	31
Abbreviated method of division of decimals, when only a certain number of decimal places are required in the quotient	35
To reduce a vulgar fraction to a decimal	33
To convert a recurring decimal into a vulgar fraction	34

CONTENTS.

vii.

X.—RATIO, PROPORTION, AND VARIATION.

	PAGE
Definitions	35
To find the ratio of one number to another	36
To simplify the terms of a ratio, when fractional	36
To compound two or more ratios	36
To compare two ratios	36
To divide a number into parts which shall bear to each other given ratios	37
To determine whether four numbers be proportionals in a given order	37
Having given three numbers, to find a fourth such, that all shall be proportional	37
To find a third proportional to two given numbers	38

XI.—INVOLUTION.

Definitions	38
To involve a number to a given power	38

XII.—EVOLUTION.

Definitions	39
To find the square root of a given number	39
To extract the cube root of a given number	41

XIII.—ARITHMETIC PROGRESSION.

Definitions	43
To find any required term of an arithmetic series, of which the first term and the common difference are known	44
To find the sum of any number of terms of a given arithmetic series	44
To find any number of arithmetic means between two numbers	44

XIV.—GEOMETRIC PROGRESSION.

Definitions	44
To find any required term of a given geometric progression.	46
To find the sum of any number of terms of a geometric series	46
To find the limit of the sum of an infinite geometric series	46
To find any number of geometric means between two numbers	46

XV.—SCALES OF NOTATION.

Observations and definitions	46
To change an integer from one scale to another	46
To change a fraction from one scale to another	46
To add, subtract, multiply, or divide in any scale	47

PART II.

COMMERCIAL ARITHMETIC.

I.—REDUCTION OF INTEGERS.

	PAGE
Definitions	48
To change numbers from a higher denomination to a lower	48
To change numbers from a lower denomination to a higher	49
To change numbers from one denomination to another, when no exact number of the one is contained in one of the other. Or, to express one quantity in terms of another	50

II.—REDUCTION OF FRACTIONS.

To express in lower terms the value of a fraction of a simple quantity	52
To express in higher terms the value of a fraction of a simple quantity	53
To express a compound quantity as a fraction of any simple quantity	54
To express shillings, pence, and farthings as decimals of a pound	54
To express in positive terms the value of a decimal of a pound	55
To express in positive terms the value of a fraction of a compound quantity	55
To express one quantity, fractional or other, in terms, or as a fraction, of another. Or, to find the ratio of one to the other	56

III.—COMPOUND ADDITION.

To add several compound quantities of the same kind together	57
To express in positive terms the sum of several fractions of concrete quantities.	58

IV.—COMPOUND SUBTRACTION.

To subtract one compound quantity from another of the same kind	59
To express in positive terms the difference of two fractions of concrete quantities	60

V.—COMPOUND MULTIPLICATION AND DIVISION.

To multiply by any number a given compound quantity	61
To divide a compound quantity by any number	63
To multiply a compound quantity by a vulgar fraction	66
To multiply a compound quantity by a decimal fraction	66
To divide a compound quantity by a vulgar fraction	67
To divide a compound quantity by a decimal fraction	67

VI.—PRACTICE.

When the quantity of goods is expressed by a simple number, the price of one being given	68
When the quantity of goods is not expressed by a simple number	68

VII.—SIMPLE PROPORTION OR RULE OF THREE.

Observations and rule	71
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