

**A SEQUEL TO THE FIRST SIX  
BOOKS OF THE ELEMENTS  
OF EUCLID, AN EASY  
INTRODUCTION TO MODERN  
GEOMETRY**

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A Sequel to the First Six Books of the Elements of Euclid, an Easy Introduction to Modern  
Geometry by John Casey

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**JOHN CASEY**

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DUBLIN UNIVERSITY PRESS SERIES.

A SEQUEL TO THE FIRST SIX BOOKS

OF THE

ELEMENTS OF EUCLID,

CONTAINING

AN EASY INTRODUCTION TO MODERN GEOMETRY.

With numerous Examples.

BY

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## PREFACE.

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I HAVE endeavoured in this Manual to collect and arrange all those elementary Geometrical Propositions not given in Euclid, which a Student will require in his Mathematical Course. The necessity for such a work will be obvious to every person engaged in Mathematical tuition. I have been frequently obliged, when teaching the Higher Mathematics, to interrupt my demonstrations, in order to prove some elementary Propositions on which they depended, but which were not given in any book to which I could refer. The object of the present little Treatise is to supply that want.

The following is the plan of the Work. It is divided into five Chapters, corresponding to Books I., II., III., IV., VI. of Euclid. The Supplements to Books I.-IV. consist of two Sections each, namely, Section I., Additional Propositions; Section II., Exercises. This part will be found to contain original proofs of some of the

most elegant Propositions in Geometry. The Supplement to Book VI. is the most important; it embraces more than half the work, and consists of eight Sections, as follows:—I., Additional Propositions; II., Centres of Similitude; III., Theory of Harmonic Section; IV., Theory of Inversion; V., Coaxal Circles; VI., Theory of Anharmonic Section; VII., Theory of Poles and Polars, and Reciprocation; VIII., Miscellaneous Exercises. Some of the Propositions in these Sections have first appeared in Papers published by myself; but the greater number have been selected from the writings of CHASLES, SALMON, and TOWNSEND. For the proofs given by those authors, in some instances others have been substituted, but in no case except where by doing so they could be made more simple and elementary.

I have to return my best thanks to MR. WILLIAMSON, F.R.S., for many suggestions, as well as for his kindness in reading the proof sheets; and also to the Committee of the "DUBLIN UNIVERSITY PRESS SERIES," for adopting my book as one of their publications.

JOHN CASEY.

2, IONA TERRACE,  
DUBLIN, Feb. 8, 1881.



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