CAMBRIDGE SENATE-HOUSE PROBLEMS AND RIDERS FOR THE YEAR 1860: WITH SOLUTIONS

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Cambridge Senate-House Problems and Riders for the Year 1860: With Solutions by H. W. Watson & E. J. Routh

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H. W. WATSON & E. J. ROUTH

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Cambridge Senate-House Problems and Riders

FOR THE YEAR 1860;

WITH SOLUTIONS.

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PREFACE.

THE value of a Collection of Solutions depends in great measure on the fact that every Problem is solved by the framer of the question, thus showing the student the manner in which he was expected to proceed in the Senate-House. The Moderators desire therefore to thank the Examiners for the many valuable Solutions of the Problems set by them, by which the book has been made more complete than it would otherwise have been.

The Senior Moderator also acknowledges his obligation to Mr DROOP, Fellow of Trinity College, for much valuable assistance, and particularly for the suggestion and the solution of the three following Problems, viz. No. vi. of Tuesday Morning, Jan. 17, and Nos. 3 and 5 of Wednesday Morning, Jan. 18.

SOLUTIONS OF SENATE-HOUSE PROBLEMS AND RIDERS

FOR THE YEAR EIGHTEEN HUNDRED AND SIXTY.

TUESDAY, *Jan.* 3. 9 *to* 12.

JUNIOR MODERATOR. Arabic numbers.
SENIOR EXAMINER. Roman numbers.

1. IF a straight line DME be drawn through the middle point M of the base of a triangle ABC, so as to cut off equal parts AD, AE from the sides AB, AC, produced if necessary, respectively, then shall BD be equal to CE.

Through C draw CF parallel to AB, and cutting DE in F. Then the two triangles DMB, FMC are clearly equal, and therefore $CF = BD$. Again, CF being parallel to AB, the angle $CFE =$ the angle ADE, and because $AD = AE$, the angle $ADE =$ angle AED; whence it easily follows that $CF = CE$.

2. Shew how to construct a rectangle which shall be equal to a given square; (1) when the sum and (2) when the difference of two adjacent sides is given.

The first case is too obvious to require any solution. In the second case, refer to the figure in Euclid, Book II. Prop. 14. A little consideration will shewthat GE is twice the difference between the two sides BE, ED. Whence the following construction. Take $GE =$ half the given difference, describe a circle BHF with radius equal to the side of the given square, and cutting GE produced in B and F. Then BE, EF are the sides of the rectangle required.

3. If two chords AB, AC be drawn from any point A of a circle, and be produced to D and E, so that the rectangle AC, AE is equal to the rectangle AB, AD, then if O be the centre of the circle, AO is perpendicular to DE.

Since $AB \cdot AD = AC \cdot AE$, a circle may be described about $BCED$. Therefore the angle $BDE = BCA$. Hence if A and B be fixed while C moves round the circle, the angle ADE will be constant and the locus of E will be a straight line. Take AC to pass through O and cut the circle in C' and DE in P. Then as before the angle $APD = ABC' =$ a right angle.

iv. Describe an isosceles triangle having each of the angles at the base double of the third angle.

If A be the vertex, and BD the base of the constructed triangle, D being one of the points of intersection of the two circles employed in the construction, and E the other, and AE be drawn meeting BD produced in F, prove that FAB is another isosceles triangle of the same kind.

For ADE is an isosceles triangle, and the angle AED at the base is the supplement of the angle ACD in the opposite segment of the circle. Hence $AED = BCD$ and therefore by Euclid $= ABD$, and also the angles ADE, ADB are equal, therefore the third angle $DAE =$ the third angle BAD. Hence the whole angle BAE is double the angle BAD, and therefore equal to ABD. Hence the triangle FAB is isosceles, and each of the angles at the base is equal to the angles at the base of ABD. Therefore, &c.

v. Prove that the straight lines bisecting one angle of a triangle internally and the other two externally pass through the same point.

Let the exterior angles A and C of the triangle ABC be bisected by AD, CO, meeting each other in O; then BO will bisect the angle ABC. Because AD bisects the exterior angle A, $BA : BD :: AC : CD$. And because CO bisects the angle ACD, therefore $AC : CD :: AO : OD$, therefore $BA : BD :: AO : OD$, and therefore BO bisects the angle ABD. See fig. 1.

vi. If three straight lines, which do not all lie in one plane, be cut in the same ratio by three planes, two of which are parallel, shew that the third will be parallel to the other two, if its intersections with the three straight lines are not all in one straight line.

This may be easily proved by a "reductio ad absurdum."

vii. Define a parabola: and prove from the definition that it cannot be cut by a straight line in more than two points.

For if possible let a straight line cut the parabola in three points P, Q, R, and let it cut the directrix in T. Draw Pp, Qq, Rr perpendiculars to the directrix, and let S be the focus. Then since $SP = Pp$, $SQ = Qq$, it follows that $SP : SQ :: PT : QT$, and therefore ST bisects the exterior angle to PSQ. Similarly ST also bisects the exterior angle to PSR. Which is absurd.

viii. P, Q are points in two confocal ellipses, at which the line joining the common foci subtends equal angles; prove that the tangents at P, Q are inclined at an angle which is equal to the angle subtended by PQ at either focus.

Let the normals at P and Q meet in G, join QP and produce it to any point R. Then the angle between the tangents is equal to the angle PGQ which is

$$= RPG - RQG = (RPS - RQS) + (SPG - SQG).$$

Now $SPG = SQG$, being the halves of equal angles, and the difference $RPS - RQS = PSQ$. Similarly the angle PGQ may be proved $= PHQ$.

ix. If a circle, passing through Y and Z, touch the major axis in Q, and that diameter of the circle, which passes through Q, meet the tangent in P, then $PQ = BC$.