# A MEMOIR OF THE THEORY OF MATHEMATICAL FORM

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A Memoir of the Theory of Mathematical Form by A. B. Kempe



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ON THE

# THEORY OF MATHEMATICAL FORM.

BY

A. B. KEMPE, M.A., F.R.S.

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# PHILOSOPHICAL TRANSACTIONS.

## I. *A Memoir on the Theory of Mathematical Form.**

*By* A. B. KEMPE, *M.A., F.R.S.*

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* In accordance with criticisms and suggestions of the referees, an alteration has been made in the title of the memoir, and certain sections towards the end have been omitted. There is consequently a slight want of complete correspondence between the memoir and the abstract given in the Proceedings, vol. 38, p. 393.—A. B. K.

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## *Scope of the Memoir.*

1. My object in this memoir is to separate the necessary matter of exact or mathematical thought from the accidental clothing—geometrical, algebraical, logical, &c.—in which it is usually presented for consideration; and to indicate wherein consists the infinite variety which that necessary matter exhibits.

2. The memoir is confined to the exposition of fundamental principles, to their elementary developments, to their application to such a variety of cases as will vindicate their value, and to a description of some simple and uniform modes of putting the necessary matter in evidence. I have been unable to ascertain that the principles here set forth have been previously formulated.

## *Fundamental Principles.*

3. Whatever may be the true nature of things and of the conceptions which we have of them (into which points we are not here concerned to inquire), in the operations of reasoning they are dealt with as a number of separate entities or *units*.

4. These units come under consideration in a variety of garbs—as material objects, intervals or periods of time, processes of thought, points, lines, statements, relationships, arrangements, algebraical expressions, operators, operations, &c., &c., occupy various positions, and are otherwise variously circumstanced. Thus, while some units are incapable of being distinguished from each other, others are by these peculiarities rendered distinguishable. For example, the angular points of a square are distinguishable from the sides, but are not distinguishable from each other. In some instances where distinctions exist they are ignored as not material. Both cases are included in the general statement that some units are distinguished from each other and some are not.

5. In like manner some *pairs* of units are distinguished from each other, while others are not. Pairs may in some cases be distinguished even though the units

composing them are not. Thus the angular points of a square are undistinguishable from each other, and a pair of such points lying at the extremities of a side are undistinguishable from the three other like pairs, but are distinguished from the two pairs formed by taking the angular points at the extremities of the diagonals, which pairs again are undistinguishable from each other. Further, a pair $ab$ may sometimes be distinguished from the pair $ba$ though the units $a$ and $b$ are undistinguished. Thus, in fig. 1 the three black spots $a$, $b$, $c$, are all undistinguished from each other, each has an

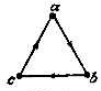


Fig. 1.

arrow proceeding to it and an arrow proceeding from it, but the pair $ab$ is distinguished from the pair $ba$, for an arrow proceeds from $a$ to $b$, but none from $b$ to $a$.

6. It will be convenient to speak of $ab$ and $ba$ as different *aspects* of the *collection* of two units $a$, $b$. Here the terms "aspect" and "collection" are each to be understood as referring to two separate units, and not to those units regarded in the aggregate as a single unit.

7. Again, there are also distinguished and undistinguished *triads*, *tetrads*, ... *m-ads*, ... *n-ads* ...; every $m$-ad being, of course, distinguished from every $n$-ad. Just as we may have $ab$ distinguished from $ba$, though $a$ is undistinguished from $b$, so we may have an $n$-ad $pqrst \ldots uv$ distinguished from $qusvt \ldots rp$, though the units $p$, $q$, $r$, $s$, &c., are all undistinguished from each other, and further, though their pairs are also undistinguished, as likewise their triads, &c. Here $pqrst \ldots uv$ and $qusvt \ldots rp$ will be termed, as in the case of pairs, different *aspects* of the *collection* $p, q, r, s, t, \ldots u, v$; the term "collection" being understood to refer to a number of separate units without reference to the various "aspects" of the collection. Different aspects of the same collection of $n$ units will be regarded as different $n$-ads.

8. The terms "pair," "triad" ... "$n$-ad," "collection," "aspect" will always be understood to refer to two, three, $n$, &c., units, and never to aggregations of units considered as a single unit. Pairs, triads, $n$-ads, collections, aspects may, of course, be regarded as units, but when they are so regarded the fact will be distinctly pointed out.

9. Every collection of units has a definite *form* due (1) to the number of its


Fig. 2.

component units, and (2) to the way in which the distinguished and undistinguished units, pairs, triads, &c., are distributed through the collection. Two collections of

the same number of units but having different distributions will be of different forms. Thus the two tetrads $a$, $b$, $c$, $d$, and $p$, $q$, $r$, $s$, of fig. 2, contain the same number (four) of unit spots, but they are of different forms; for $a$, $b$, $c$, $d$ are all undistinguished from each other; while $q$, $r$, $s$, though undistinguished from each other, are all distinguished from $p$. The distribution of the distinguished and undistinguished pairs and triads is also obviously different in the two cases. The word "form" will in this memoir be always employed in the sense here indicated.

10. Two collections of units which are undistinguished are of the same form, but two collections which are of the same form are not necessarily undistinguished; there may be the same distribution of distinguished and undistinguished units, pairs, &c., in each, while the units, pairs, &c., of one are all distinguished from the units, pairs, &c., of the other.

11. Each of the forms which a system of any number $n$ of units can assume, owing to varieties of distribution, is one of a definite number of possible forms, and the peculiarities and properties of the collection depend, as far as the processes of reasoning are concerned, upon the particular form it assumes, and are independent of the dress, geometrical, algebraical, logical, &c., in which it is presented; so that two systems which are of the same form have precisely the same properties, although the garbs in which they are severally clothed may, by their dissimilarity, lead us to place the systems under very different categories, and even to regard them as belonging to "different branches of science."

12. It may seem in some cases that other considerations are involved besides "form," but it will be found on investigation that the introduction of such considerations involves also the introduction of fresh units, and then we have merely to consider the form of the enlarged collection.

13. In order to put form in evidence some "accidental" clothing is of course necessary; if, however, we employ more than one species of clothing, each species being uniform and suited to forms of every kind, the likelihood of its accidental nature being overlooked will be reduced to a minimum.

### *Units.*

14. The units which we have to consider exhibit endless variety; thus we may have a material object dealt with as one unit, a quality it possesses as another, a statement about it as a third, and a position it occupies in space as a fourth. The task of specifying the units which are considered in an investigation may in some cases be one of considerable difficulty, and mistakes are likely to occur unless the operation is conducted with great care.

15. We have frequently to deal with things $x$, $y$, $z$, &c., pairs, &c., of those things, the differences between which depend on the existence or non-existence of certain circumstances, or upon taking into account or ignoring certain circumstances. Thus we may apparently have collections of units $x$, $y$, $z$, &c., which are at one time $t$, when

one state of things exists, of one form, and at another time $t'$ of another form. This is, however, not so; the form of a collection is absolutely invariable. The apparent alteration of form arises from overlooking the fact that the units dealt with are not $x, y, z$, &c., but ($x$ at time $t$), ($y$ at time $t$), &c., ($x$ at time $t'$), ($y$ at time $t'$), &c. The unit ($x$ at time $t$) may be quite different from ($x$ at time $t'$). Where there is only one alternative $t$, the collection ($x$ at time $t$), ($y$ at time $t$), ($z$ at time $t$), &c., is of the same form as the collection $x, y, z$, &c., and it matters not which is the collection dealt with.

16. Aggregations of those things which, in a symbolical representation of the units considered in any case, are already represented as units, must not be supposed to be sufficiently represented without additional symbols, but must each be represented by a distinct symbolical unit. An aggregation of things is, as far as the processes of reasoning are concerned, a mere unit, and must be so dealt with.

17. While it is important that a unit should be represented and dealt with as a unit, it is equally important that we should not be misled by our modes of thought and consequent use of language into regarding a number of distinct units as one only. A collection of units must in a symbolical representation be represented as a single unit where it is so regarded; but where the word "collection" is used, as it is here (sec. 8), merely to "denote" or mark off a number of things each of which is considered as a distinct unit, we must be careful to represent each of those things by a distinct symbol.

*Some Definitions.*

18. Any collection of units which consists entirely of units selected from another collection will be termed a *component* of the latter. Any aspect of a component of a collection may also be spoken of as a component of the collection.

19. Two collections of units will be said to be *detached* if they have no component in common.

20. An $n$-ad which has one or more units in common with each of a number of collections will be said to be an $n$-ad *connecting* those collections; *e.g.*, the pairs which a single unit makes with the various units of a collection will be termed the pairs connecting the unit and the collection. An $n$-ad connecting $n$ detached collections has one unit and one only in common with each. If A, B, C, D, .... be collections of which $a, b, c, d, \ldots$ are component units respectively; when an $n$-ad connecting A B C D ... is spoken of, it must be understood that an $n$-ad such as $a\ b\ c\ d \ldots$ is meant, and not one such as $b\ d\ c\ a \ldots$, which will be spoken of as connecting B D C A ...

21. If the component units of a collection are all undistinguished from each other the collection will be said to be *single*.

22. Units which are undistinguished from the same unit are undistinguished from each other; thus if a collection of units is not single it consists of two or more detached single collections, and will be termed a *double, treble, &c.*, collection, according to the number of component single collections which it contains.