

**AMERICAN MATHEMATICAL SOCIETY
COLLOQUIUM LECTURES, VOLUME V:
THE CAMBRIDGE COLLOQUIUM, 1916.
PART I. FUNCTIONALS AND THEIR
APPLICATIONS; SELECTED TOPICS,
INCLUDING INTEGRAL EQUATIONS**

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American Mathematical Society Colloquium Lectures, Volume V: the Cambridge Colloquium, 1916. Part I. Functionals and Their Applications; Selected Topics, Including Integral Equations
by Griffith Conrad Evans

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1916

PART I

**FUNCTIONALS AND THEIR APPLICATIONS
SELECTED TOPICS, INCLUDING
INTEGRAL EQUATIONS**

BY

GRIFFITH CONRAD EVANS

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1918

TO
MY FATHER
GEORGE WILLIAM EVANS
THESE LECTURES ARE AFFECTIONATELY
DEDICATED

PREFACE

The American Mathematical Society held its Eighth Colloquium in connection with the Twenty-Third Summer Meeting at Harvard University during the week of September 3-9, 1916. At this Colloquium the following lectures were delivered:

I. Functionals and their Applications. Selected Topics, including Integral Equations. By Professor Griffith C. Evans of the Rice Institute.

II. Analysis Situs. By Professor Oswald Veblen of Princeton University.

The present volume, which is issued as Part I, contains Professor (now Captain) Evans's lectures. Professor (now Captain) Veblen has been prevented by national service from preparing his manuscript for publication. The Committee hopes that, in the not too distant future, his lectures may appear as Part II of a single volume. It seems best, however, to issue Professor Evans's lectures promptly, even though a certain discontinuity may thereby result.

LUTHER P. EISENHART,
WILLIAM F. OSGOOD,
R. G. D. RICHARDSON,
Committee on Publication.

AUTHOR'S PREFACE.

Most mathematicians are familiar with that development of the subject of integral equations which is epitomized by the name Hilbert. There are however other domains whose description is not so completely available in book form, which represent nevertheless an expansion of the same circle of fundamental notions, implied in the central theory of integral equations. Volterra's genial concepts, developed during the last thirty years, and outlined in his treatise of 1913, have by contact with the ideas of Hadamard, Stieltjes, Lebesgue, Borel and others, given rise to many new points of view. It is the purpose of the Lectures to select for discussion some of those which appear most to promise further rapid expansion.

A word may be necessary as to the arrangement. In order to make the subject matter accessible to as large a circle of readers as possible, the text in large type has been devised to be intelligible to those who approach the subject for the first time, and may be read by itself. The text in small type comes, on the other hand, closer to the present state of the subject, and may be more suggestive. The author thus hopes to fulfil the avowed purpose of the Colloquium.

Commas in formulæ are omitted when not necessary for clearness; thus $r(\lambda, t)$ is written $r(\lambda t)$ if the meaning is clear from the context.

The author has given references to American mathematicians freely, in order that familiarity with names may stimulate conversation at meetings of the Society, and thereby increase interest in the subject itself.

HOUSTON, TEXAS,
February, 1918.

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