FIVE-PLACE LOGARITHMIC AND TRIGONOMETRIC TABLES

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G. A. HILL & G. A. WENTWORTH

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ARRANGED BY

G. A. WENTWORTH, A.M.,

AND

G. A. HILL, A.M.

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INTRODUCTION.

1. If the natural numbers are regarded as powers of ten, the exponents of the powers are the Common or Briggs Logarithms of the numbers. If A and B denote natural numbers, a and b their logarithms, then $10^a = A$, $10^b = B$; or, written in logarithmic form,

$$\log A = a, \qquad \log B = b.$$

2. The logarithm of a product is found by adding the logarithms of its factors.

For, $\qquad A \times B = 10^a \times 10^b = 10^{a+b}.$
Therefore, $\qquad \log (A \times B) = a + b = \log A + \log B.$

3. The logarithm of a quotient is found by subtracting the logarithm of the divisor from that of the dividend.

For, $\qquad \frac{A}{B} = \frac{10^a}{10^b} = 10^{a-b}.$

Therefore, $\qquad \log \frac{A}{B} = a - b = \log A - \log B.$

4. The logarithm of a power of a number is found by multiplying the logarithm of the number by the exponent of the power.

For, $\qquad A^n = (10^a)^n = 10^{an}.$
Therefore, $\qquad \log A^n = an = n \log A.$

5. The logarithm of the root of a number is found by dividing the logarithm of the number by the index of the root.

For, $\qquad \sqrt[n]{A} = \sqrt[n]{10^a} = 10^{\frac{a}{n}}.$

Therefore, $\qquad \log \sqrt[n]{A} = \frac{a}{n} = \frac{\log A}{n}.$

6. The logarithms of 1, 10, 100, etc., and of 0.1, 0.01, 0.001, etc., are integral numbers. The logarithms of all other numbers are fractions.

For, $10^0 = 1$, hence $\log 1 = 0$; $10^{-1} = 0.1$, hence $\log 0.1 = -1$;
$10^1 = 10$, hence $\log 10 = 1$; $10^{-2} = 0.01$, hence $\log 0.01 = -2$;
$10^2 = 100$, hence $\log 100 = 2$; $10^{-3} = 0.001$, hence $\log 0.001 = -3$;
$10^3 = 1000$, hence $\log 1000 = 3$; and so on.

If the number is between 1 and 10, the logarithm is between 0 and 1.
If the number is between 10 and 100, the logarithm is between 1 and 2.
If the number is between 100 and 1000, the logarithm is between 2 and 3.
If the number is between 1 and 0.1, the logarithm is between 0 and −1.
If the number is between 0.1 and 0.01, the logarithm is between −1 and −2.
If the number is between 0.01 and 0.001, the logarithm is between −2 and −3.
And so on.

7. If the number is less than 1, the logarithm is negative (§ 6), but is written in such a form that the *fractional part* is always *positive*.

For the number may be regarded as the product of two factors, one of which lies between 1 and 10, and the other is a negative power of 10; the logarithm will then take the form of a *difference* whose minuend is a positive proper fraction, and whose subtrahend is a positive integral number.

Thus, $0.48 = 4.8 \times 0.1$.
Therefore (§ 2), $\log 0.48 = \log 4.8 + \log 0.1 = 0.68124 - 1$. (Page 1.)
Again, $0.0007 = 7 \times 0.0001$.
Therefore, $\log 0.0007 = \log 7 + \log 0.0001 = 0.84510 - 4$.

8. Every logarithm, therefore, consists of two parts: a positive or negative integral number, which is called the **Characteristic**, and a *positive* proper fraction, which is called the **Mantissa**.

Thus, in the logarithm 3.52179, the integral number 3 is the characteristic, and the fraction .52179 the mantissa. In the logarithm $0.78254 - 2$, the integral number -2 is the characteristic, and the fraction .78254 is the mantissa.

9. If the logarithm is negative, it is customary to change the form of the difference so that the subtrahend shall be 10 or a multiple of 10. This is done by adding to both minuend and subtrahend a number which will increase the subtrahend to 10 or a multiple of 10.

Thus, the logarithm $0.78254 - 2$ is changed to $8.78254 - 10$ by adding 8 to both minuend and subtrahend. The logarithm $0.92737 - 13$ is changed to $7.92737 - 20$ by adding 7 to both minuend and subtrahend.

10. The following rules are derived from § 6: —

If the number is *greater than* 1, make the characteristic of the logarithm *one unit less* than the number of figures on the left of the decimal point.

If the number is *less than* 1, make the characteristic of the logarithm *negative*, and *one unit more* than the number of zeros between the decimal point and the first significant figure of the given number.

If the characteristic of a given logarithm is *positive*, make the number of figures in the integral part of the corresponding number *one more* than the number of units in the characteristic.

If the characteristic is *negative*, make the number of zeros between the decimal point and the first significant figure of the corresponding number *one less* than the number of units in the characteristic.

Thus, the characteristic of $\log 7849.27 = 3$;
the characteristic of $\log 0.037 = -2 = 8.00000 - 10$.
If the characteristic is 4, the corresponding number has five figures in its integral part. If the characteristic is -3, that is, $7.00000 - 10$, the corresponding fraction has two zeros between the decimal point and the first significant figure.

11. The logarithms of numbers that can be derived from one another by multiplication or division by an integral power of 10 have the same mantissa.

For, multiplying or dividing a number by an integral power of 10 will increase or diminish its logarithm by the exponent of that power of 10; and since this exponent is an integer, the mantissa of the logarithm will be unaffected.

Thus,

$$\log 4.6021 = 0.66296. \quad \text{(Page 9.)}$$

$$\log 460.21 = \log(4.6021 \times 10^2) = \log 4.6021 + \log 10^2 = 0.66296 + 2 = 2.66296.$$

$$\log 460210 = \log(4.6021 \times 10^5) = \log 4.6021 + \log 10^5 = 0.66296 + 5 = 5.66296.$$

$$\log 0.046021 = \log(4.6021 \div 10^2) = \log 4.6021 - \log 10^2 = 0.66296 - 2 = 8.66296 - 10.$$

TABLE I.

12. In this table (pp. 1–19) the vertical columns headed N contain the numbers, and the other columns the logarithms. On page 1 both the characteristic and the mantissa are printed. On pages 2–19 the mantissa only is printed.

The fractional part of a logarithm can be expressed only approximately, and in a five-place table all figures that follow the fifth are rejected. Whenever the sixth figure is 5, or more, the fifth figure is increased by 1. The figure $\underline{5}$ is written when the value of the figure in the place in which it stands, together with the succeeding figures, is more than $4\frac{1}{2}$, but less than 5.

Thus, if the mantissa of a logarithm written to seven places is 5328732, it is written in this table (a five-place table) 53287. If it is 5328751, it is written 53288. If it is 5328461 or 5328499, it is written in this table 5328$\underline{5}$.

Again, if the mantissa is 5324981, it is written 532$\underline{5}$0; and if it is 4999967, it is written $\underline{5}$0000.

This distinction between 5 and $\underline{5}$, in case it is desired to curtail still further the mantissas of logarithms, removes all doubt whether a 5 in the last given place, or in the last but one followed by a zero, should be simply rejected, or whether the rejection should lead us to increase the preceding figure by one unit.

Thus, the mantissa 1392$\underline{5}$ when reduced to four places should be 1392; but 13925 should be 1393.

To Find the Logarithm of a Given Number.

13. If the given number consists of one or two significant figures, the logarithm is given on page 1. If zeros follow the significant figures, or if the number is a proper decimal fraction, the characteristic must be determined by § 10.

14. If the given number has three significant figures, it will be found in the column headed N (pp. 2–19), and the mantissa of its logarithm in the next column to the right, and on the same line. Thus,

Page 2. $\log 145 = 2.16137$, $\log 14500 = 4.16137$.
Page 14. $\log 716 = 2.85491$, $\log 0.716 = 9.85491 - 10$.

15. If the given number has four significant figures, the first three will be found in the column headed N, and the fourth at the top of the page in the line containing the figures **1, 2, 3,** etc. The mantissa will be found in the column headed by the fourth figure, and on the same line with the first three figures. Thus,

Page 15. $\log 7682 = 3.88547$, $\log 76.85 = 1.88564$.
Page 18. $\log 93280 = 4.96979$, $\log 0.9468 = 9.97626 - 10$.

16. If the given number has five or more significant figures, a process called **interpolation** is required.

Interpolation is based on the *assumption* that between two consecutive mantissas of the table the change in the mantissa is directly proportional to the change in the number.

Required the logarithm of 34237.

The required mantissa is (§ 11) the same as the mantissa for 3423.7; therefore it will be found by adding to the mantissa of 3423 seven-tenths of the difference between the mantissas for 3423 and 3424.

The mantissa for 3423 is 53441.

The difference between the mantissas for 3423 and 3424 is 12.

Hence, the mantissa for 3423.7 is $53441 + (0.7 \times 12) = 53449$.

Therefore, the required logarithm of 34237 is 4.53449.

Required the logarithm of 0.0015764.

The required mantissa is the same as the mantissa for 1576.4; therefore it will be found by adding to the mantissa for 1576 four-tenths of the difference between the mantissas for 1576 and 1577.

The mantissa for 1576 is 19756.

The difference between the mantissas for 1576 and 1577 is 27.

Hence, the mantissa for 1576.4 is $19756 + (0.4 \times 27) = 19767$.

Therefore, the required logarithm of 0.0015764 is $7.19767 - 10$.

Required the logarithm of 32.6708.

The required mantissa is the same as the mantissa for 3267.08; therefore it will be found by adding to the mantissa for 3267 eight-hundredths of the difference between the mantissas for 3267 and 3268.

The mantissa for 3267 is 51415.

The difference between the mantissas for 3267 and 3268 is 13.

Hence, the mantissa for 3267.08 is $51415 + (0.08 \times 13) = 51416$.

Therefore, the required logarithm of 32.6708 is 1.51416.

17. When the fraction of a unit in the part to be added to the mantissa for four figures is less than 0.5 it is to be neglected; when it is 0.5 or more than 0.5 it is to be taken as one unit.

Thus, in the first example, the part to be added to the mantissa for 3423 is 8.4, and the .4 is rejected. In the second example, the part to be added to the mantissa for 1576 is 10.8, and 11 is added.

To Find the Number Corresponding to a Given Logarithm.

18. If the given mantissa can be found in the table, the first three figures of the required number will be found in the same line with the mantissa in the column headed N, and the fourth figure at the top of the column containing the mantissa.

The position of the decimal point is determined by the characteristic (§ 10).

Find the number corresponding to the logarithm 0.92002.

Page 16. The number for the mantissa 92002 is 8318.

Therefore, the required number is 8.318.

Find the number corresponding to the logarithm 6.09167.

Page 2. The number for the mantissa 09167 is 1235.

Therefore, the required number is 1235000.

Find the number corresponding to the logarithm $7.50325 - 10$.

Page 6. The number for the mantissa 50325 is 3186.

Therefore, the required number is 0.003186.