SOLUTIONS OF GOODWIN'S COLLECTION OF PROBLEMS AND EXAMPLES, PP. 2-121

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OF

GOODWIN'S COLLECTION

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PROBLEMS AND EXAMPLES.

BY THE

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PREFACE.

The Problems and Examples which I published as a companion to my Course of Mathematics were, with a few exceptions, unaccompanied by solutions; one principal reason for adopting that method was, that it seemed to be a better exercise for a student to attempt the solution of a problem without the assistance which may be derived from inspection of the method to be adopted, or perhaps even from mere knowledge of the result. This opinion I still hold; and I shall be sorry if the present volume be found to impair the usefulness of the other, a result which *must* ensue if its pages be consulted before the student has fully exercised his own powers in attempts at independent solution of the problems.

A volume of solutions of my Collection of Problems and Examples being much demanded, and my own engagements rendering it impossible for me to undertake the work, I entrusted the execution of it to the Rev. W. W. Hutt, Fellow and Sadlerian Lecturer of Caius College. That gentleman kindly undertook the office, and has prepared the work for publication as rapidly as the pressure of other engagements would allow. Mr Hutt has been assisted in a considerable portion of the work by Mr P. H. Mason, B.A., Scholar of St John's College.

The references throughout the following pages are to the *second edition* of my Course of Mathematics.

H. GOODWIN.

CAMBRIDGE,
May, 1849.

Similarly, $a^4 + b^4 + c^4 = 98 = 14\,(a + 2c)$,

$$a^5 + b^5 = 33 = 11c,$$

$$a^2 - ab + b^2 = 3 = c,$$

and $(a + b)\,(a + c)\,(b + c) = 60 = 10abc$.

8. Answers. 182, 64, 48, -16, 7245, 3720.

9. When $x = 3$,

$$x^3 - 3x^2 + 3x - 1 = 27 - 27 + 9 - 1 = 8.$$

10. When $x = \frac{1}{12}$,

$$\sqrt{\tfrac{3}{4} - x} + \sqrt{2x} - \tfrac{3}{2}\sqrt{1 - 4x} = \sqrt{\tfrac{3}{4} - \tfrac{1}{12}} + \sqrt{\tfrac{1}{6}} - \tfrac{3}{2}\sqrt{1 - \tfrac{1}{3}}$$

$$= \sqrt{\tfrac{8}{12}} + \sqrt{\tfrac{1}{6}} - 3\sqrt{\tfrac{1}{4} \cdot \tfrac{2}{3}} = 2\sqrt{\tfrac{1}{6}} + \sqrt{\tfrac{1}{6}} - 3\sqrt{\tfrac{1}{6}} = 0.$$

11. Ans. $(x + y)^{\frac{3}{2}}$, $ax^{\frac{1}{2}} + b^{\frac{2}{3}}x^{\frac{1}{3}} + c^{\frac{3}{5}} . x^{\frac{1}{5}}$, $a^{\frac{1}{3}}$, $a^{\frac{1}{2}} . b^{\frac{1}{4}} . c^{\frac{1}{3}}$.

ADDITION.

1. Ans. $4x^2 + 3ax + 2a^2$.

2. ... $4x^m + 5ax^{m-1} + 4a^2x^{m-2} - 5a^m$.

3. ... $4a + 4b - 3c + d$.

4. ... $6a - d$.

5. ... $4a^3 - a^2b - 2ab^2$.

6. ... $2ab + 2ac - 2ad + 2bd - 2bc + 2cd$.

7. ... $2a^2b - a^2c + a^2d + b^2c + b^2d - abc$.

8. ... $(6a - 4c)\,x$.

9. ... $2ax$.

10. ... $(2x - 5y)\,y$.

11. ... $(7a + 3b)\,x - (a - 6b)\,y$.

12. ... $4a - 2b - 2c + 2d$.

13. Ans. $3a^3 + a^2b + ac^2 + ab^2 - b^2c$.

14. ... $5a^4 + a^2b^2 + b^4$.

15. ... $a^2b^2c^2 + 4ab^3c^2 + abc^4 + 6a^2b^4 - b^6$.

16. ... $3a^2 + 6b^2$.

17. ... $203a^3 - 157a^2b + 60ab^2 + 9b^3$.

18. ... $175a^4 - 21a^3b + 19a^2b^2 + 20ab^3 - 8b^4$.

SUBTRACTION.

1. Ans. $a + 2b - 2c + 4d$.

2. ... $-a + b + 5c$.

3. ... $ay + 4by + 2ax - 2bx$.

4. ... $2a^2x + 2aby - 3b^2x$.

5. ... $3a^3 + a^2x + 14ax^2 + 5x^3$.

6. ... $8ax + 53ay - 42bx + 2by$.

7. ... $6a^{\frac{1}{2}}b^{\frac{1}{2}} - a^{\frac{3}{2}} + 5b^{\frac{3}{2}}$.

8. ... $3a - 11b$.

9. ... $-(a - d)(x^3 - y^3) - (b - c)(x^2y - xy^2)$.

10. ... $(a - b + d)x^2y - (a + b - c)x^3 - (a - b + c)y^3$.

11. ... $4y^3 - 9xy^2$.

12. ... $a + 4b - 3c + 2d$.

13. ... $3a^3 + 4a^2b - 7ab^2$.

14. ... $2a^4 + 3a^3b + a^2b^2 + ab^3$.

15. ... $-10a^4b + a^3b^2 + 2a^2b^3 + 3ab^4 + 2b^5$.

16. ... $2a^4 - 4ab^3 + 6b^4$.

17. ... $-2b^2 - 2c^2 + 2d^2$.

18. ... $22a^3 + 6a^2b - 40ab^2 + 50b^3$.

MULTIPLICATION.

1. Ans. $a^6 x^6 y^6$.

2. ... $a^{\frac{5}{6}} x^{\frac{5}{6}}$.

3. ... $a^4 - b^2 x^2 + 2bcx^3 - c^2 x^4$.

4. ... $a^3 + b^3 + c^3 - a^2 b + 3a^2 c - ab^2 + 3ac^2 - 2abc$
$- b^2 c - bc^2$.

5. ... $x^7 + y^7$.

6. ... $2a^7 b - 5a^6 b^2 - 11a^5 b^3 + 5a^4 b^4 - 26a^3 b^5 + 7a^2 b^6$
$- 12ab^7$.

7. ... $x^6 - 2x^5 y + 2x^4 y^2 - 4x^3 y^3 + 8x^2 y^4 + 16xy^5 - 32y^6$.

8. ... $x^4 - 3x^3 + 6x^2 - 5x + 3$.

9. ... $x^{\frac{4}{3}} + 2x - 2x^{\frac{2}{3}} + 7x^{\frac{1}{3}} - 2$.

10. ... $x - x^{\frac{m}{m+n}} . y^{\frac{m-n}{m+n}} + (xy)^{\frac{n}{m+n}} + x^{\frac{2m-n}{m+n}} . y^{\frac{m-n}{m+n}}$
$- (xy)^{\frac{2m-2n}{m+n}} + x^{\frac{m-n}{m+n}} . y^{\frac{m}{m+n}} + (xy)^{\frac{m}{m+n}} - x^{\frac{m-n}{m+n}} . y^{\frac{2m-n}{m+n}} + y$.

11. ... $a^3 + b^3 + c^3 - 3abc$.

12. ... $2a^8 b^2 - a^8 bc - a^7 bc^2 - a^6 b^2 c^2 + a^7 b^2 c$.

13. ... $x^3 - 6x^2 + 11x - 6$.

14. The product of two quantities of the forms $a + b$ and $a - b$, where a and b are quantities containing more than one term, may be readily found by the formula

$$(a + b) . (a - b) = a^2 - b^2.$$

In the example the product of the first two factors

$$= \{(x^2+1) - 2x\} . \{(x^2+1) + 2x\} = (x^2+1)^2 - 4x^2 = x^4 - 2x^2 + 1,$$

and the product of all three is

$$\therefore = \{(x^4 + 1) - 2x^2\} \{(x^4 + 1) + 2x^2\} = (x^4 + 1)^2 - 4x^4$$
$$= x^8 - 2x^4 + 1 = (x^4 - 1)^2.$$