

**SOLUTIONS OF
GOODWIN'S COLLECTION
OF PROBLEMS AND
EXAMPLES, PP. 2-121**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649498000

Solutions of Goodwin's Collection of Problems and Examples, pp. 2-121 by William Wayman Hutt

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

WILLIAM WAYMAN HUTT

**SOLUTIONS OF
GOODWIN'S COLLECTION
OF PROBLEMS AND
EXAMPLES, PP. 2-121**

SOLUTIONS
OF
GOODWIN'S COLLECTION
OF
PROBLEMS AND EXAMPLES.

BY THE
REV. WILLIAM WAYMAN HUTT, M.A.,
FELLOW AND SADLERIAN LECTURER OF GONVILLE
AND CAIUS COLLEGE.



CAMBRIDGE: JOHN DEIGHTON.
LONDON: SIMPKIN, MARSHALL & CO., AND GEORGE BELL.
LIVERPOOL: DEIGHTON & LAUGHTON.

M.DCCO,XLIX.

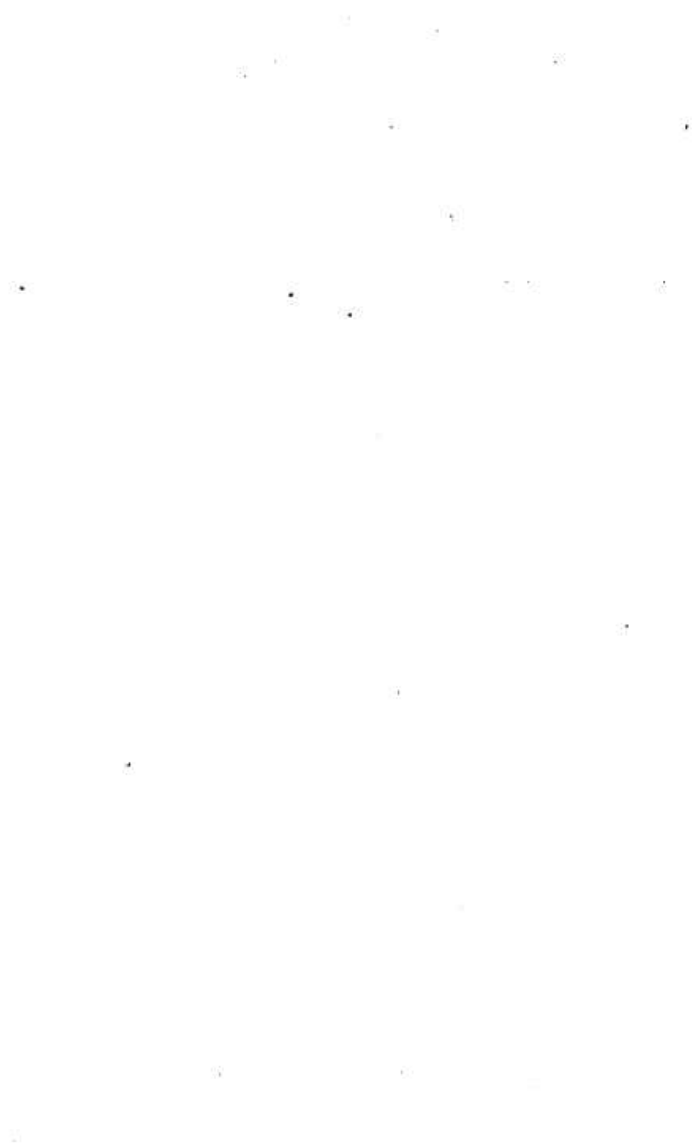


Figure 1: A scatter plot showing the relationship between the number of children (X-axis, 0 to 10) and the number of books (Y-axis, 0 to 100). The data points are: (0, 0), (1, 10), (2, 20), (3, 30), (4, 40), (5, 50), (6, 60), (7, 70), (8, 80), (9, 90), (10, 100). A straight line of best fit is drawn through the points, starting at the origin (0,0) and ending at (10,100).

PREFACE.

THE Problems and Examples which I published as a companion to my Course of Mathematics were, with a few exceptions, unaccompanied by solutions; one principal reason for adopting that method was, that it seemed to be a better exercise for a student to attempt the solution of a problem without the assistance which may be derived from inspection of the method to be adopted, or perhaps even from mere knowledge of the result. This opinion I still hold; and I shall be sorry if the present volume be found to impair the usefulness of the other, a result which *must* ensue if its pages be consulted before the student has fully exercised his own powers in attempts at independent solution of the problems.

A volume of solutions of my Collection of Problems and Examples being much demanded, and my own engagements rendering it impossible for me to undertake the work, I entrusted the execution of it to the Rev. W. W. Hutt, Fellow and Sadlerian Lecturer of Caius College. That gentleman kindly undertook the office,

and has prepared the work for publication as rapidly as the pressure of other engagements would allow. Mr Hutt has been assisted in a considerable portion of the work by Mr P. H. Mason, B.A., Scholar of St John's College.

The references throughout the following pages are to the *second edition* of my Course of Mathematics.

H. GOODWIN.

CAMBRIDGE,
May, 1849.

Similarly, $a^4 + b^4 + c^4 = 98 = 14(a + 2c)$,

$$a^5 + b^5 = 33 = 11c,$$

$$a^3 - ab + b^3 = 3 = c,$$

and $(a + b)(a + c)(b + c) = 60 = 10abc$.

8. Answers. 182, 64, 48, -16, 7245, 3720.

9. When $x = 3$,

$$x^3 - 3x^2 + 3x - 1 = 27 - 27 + 9 - 1 = 8.$$

10. When $x = \frac{1}{12}$,

$$\begin{aligned} \sqrt{\frac{3}{4} - x} + \sqrt{2x} - \frac{3}{2}\sqrt{1 - 4x} &= \sqrt{\frac{3}{4} - \frac{1}{12}} + \sqrt{\frac{1}{6}} - \frac{3}{2}\sqrt{1 - \frac{1}{3}} \\ &= \sqrt{\frac{8}{12}} + \sqrt{\frac{1}{6}} - 3\sqrt{\frac{1}{4} \cdot \frac{2}{3}} = 2\sqrt{\frac{1}{6}} + \sqrt{\frac{1}{6}} - 3\sqrt{\frac{1}{6}} = 0. \end{aligned}$$

11. Ans. $(x + y)^{\frac{3}{2}}$, $ax^{\frac{3}{2}} + b^{\frac{3}{2}}x^{\frac{3}{2}} + c^{\frac{3}{2}} \cdot x^{\frac{3}{2}}$, $a^{\frac{3}{2}}$, $a^{\frac{3}{2}} \cdot b^{\frac{1}{2}} \cdot c^{\frac{1}{2}}$.

ADDITION.

1. Ans. $4x^3 + 3ax + 2a^2$.
2. ... $4x^m + 5ax^{m-1} + 4a^2x^{m-2} - 5a^m$.
3. ... $4a + 4b - 3c + d$.
4. ... $6a - d$.
5. ... $4a^3 - a^2b - 2ab^2$.
6. ... $2ab + 2ac - 2ad + 2bd - 2bc + 2cd$.
7. ... $2a^2b - a^2c + a^2d + b^2c + b^2d - abc$.
8. ... $(6a - 4c)x$.
9. ... $2ax$.
10. ... $(2x - 5y)y$.
11. ... $(7a + 3b)x - (a - 6b)y$.
12. ... $4a - 2b - 2c + 2d$.

13. Ans. $3a^3 + a^2b + ac^2 + ab^2 - b^3c$.
 14. ... $5a^4 + a^2b^2 + b^4$.
 15. ... $a^2b^2c^2 + 4ab^2c^2 + abc^4 + 6a^2b^4 - b^6$.
 16. ... $3a^2 + 6b^2$.
 17. ... $203a^3 - 157a^2b + 60ab^2 + 9b^3$.
 18. ... $175a^4 - 21a^3b + 19a^2b^2 + 20ab^3 - 8b^4$.

SUBTRACTION.

1. Ans. $a + 2b - 2c + 4d$.
 2. ... $-a + b + 5c$.
 3. ... $ay + 4by + 2ax - 2bx$.
 4. ... $2a^2x + 2aby - 3b^2x$.
 5. ... $3a^3 + a^2x + 14ax^2 + 5x^3$.
 6. ... $8ax + 53ay - 42bx + 2by$.
 7. ... $6a^3b^3 - a^3 + 5b^3$.
 8. ... $3a - 11b$.
 9. ... $-(a-d)(x^2 - y^2) - (b-c)(x^2y - xy^2)$.
 10. ... $(a-b+d)x^2y - (a+b-c)x^3 - (a-b+c)y^3$.
 11. ... $4y^3 - 9xy^2$.
 12. ... $a + 4b - 3c + 2d$.
 13. ... $3a^3 + 4a^2b - 7ab^2$.
 14. ... $2a^4 + 3a^3b + a^2b^2 + ab^3$.
 15. ... $-10a^4b + a^3b^2 + 2a^2b^3 + 3ab^4 + 2b^5$.
 16. ... $2a^4 - 4ab^3 + 6b^4$.
 17. ... $-2b^2 - 2c^2 + 2d^2$.
 18. ... $22a^3 + 6a^2b - 40ab^2 + 50b^3$.

MULTIPLICATION.

1. Ans. $a^5 x^6 y^5$.
2. ... $a^4 x^4$.
3. ... $a^3 - b^3 x^2 + 2bcx^3 - c^2 x^4$.
4. ... $a^3 + b^3 + c^3 - a^2 b + 3a^2 c - ab^3 + 3ac^2 - 2abc$
 $- b^2 c - bc^2$.
5. ... $x^7 + y^7$.
6. ... $2a^7 b - 5a^6 b^2 - 11a^5 b^3 + 5a^4 b^4 - 26a^3 b^5 + 7a^2 b^6$
 $- 12ab^7$.
7. ... $x^5 - 2x^3 y + 2x^4 y^2 - 4x^2 y^3 + 8x^2 y^4 + 16xy^5 - 32y^6$.
8. ... $x^4 - 3x^3 + 6x^2 - 5x + 3$.
9. ... $x^{\frac{1}{2}} + 2x - 2x^{\frac{3}{2}} + 7x^{\frac{5}{2}} - 2$.
10. ... $x - x^{\frac{m}{m+n}} \cdot y^{\frac{m-n}{m+n}} + (xy)^{\frac{n}{m+n}} + x^{\frac{2m-n}{m+n}} \cdot y^{\frac{m-n}{m+n}}$
 $- (xy)^{\frac{2m-2n}{m+n}} + x^{\frac{m-n}{m+n}} \cdot y^{\frac{m}{m+n}} + (xy)^{\frac{m}{m+n}} - x^{\frac{m-n}{m+n}} \cdot y^{\frac{2m-n}{m+n}} + y$.
11. ... $a^3 + b^3 + c^3 - 3abc$.
12. ... $2a^5 b^2 - a^5 bc - a^7 bc^2 - a^6 b^2 c^2 + a^7 b^2 c$.
13. ... $x^5 - 6x^3 + 11x - 6$.

14. The product of two quantities of the forms $a + b$ and $a - b$, where a and b are quantities containing more than one term, may be readily found by the formula

$$(a + b) \cdot (a - b) = a^2 - b^2.$$

In the example the product of the first two factors

$$= \{(x^2 + 1) - 2x\} \cdot \{(x^2 + 1) + 2x\} = (x^2 + 1)^2 - 4x^2 = x^4 - 2x^2 + 1,$$

and the product of all three is

$$\therefore = \{(x^4 + 1) - 2x^2\} \cdot \{(x^4 + 1) + 2x^2\} = (x^4 + 1)^2 - 4x^4$$

$$= x^8 - 2x^4 + 1 = (x^4 - 1)^2.$$